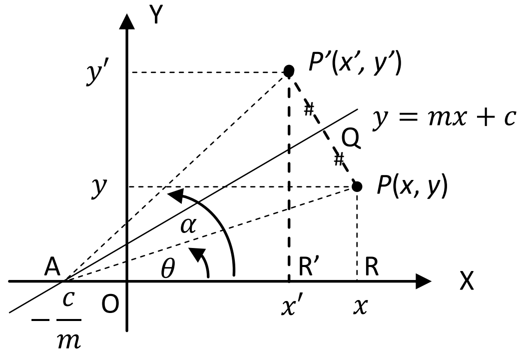
Transformasi – Refleksi Terhadap Garis $y = mx + c$



Titik $P(x, y)$ direfleksikan (dicerminkan) terhadap garis $y = mx + c$, $m \neq 0$ menghasilkan bayangan titik $P'(x', y')$.

Dalam segitiga APR berlaku

$\cos\theta = \frac{AR}{AP} \rightarrow AR = AP \cdot \cos\theta \rightarrow x + \frac{c}{m} = AP \cdot \cos\theta$ .... (1)

$\sin\theta = \frac{PR}{AP} \rightarrow PR = AP \cdot \sin\theta \rightarrow y - 0 = AP \cdot \sin\theta$ .... (2)

Dalam segitiga AP’R’ berlaku

$$\cos(2\alpha - \theta) = \frac{AR'}{AP'} = \frac{x' + \frac{c}{m}}{AP'} \rightarrow x' + \frac{c}{m} = AP' \cdot \cos(2\alpha - \theta)$$

$$\rightarrow x' + \frac{c}{m} = AP' \cdot \cos 2\alpha \cos\theta + AP' \cdot \sin 2\alpha \sin\theta$$

$$\rightarrow x' + \frac{c}{m} = \left(x + \frac{c}{m}\right) \cdot \cos 2\alpha + y \cdot \sin 2\alpha$$

$$\rightarrow \quad x' = x \cdot cos\, 2\alpha + \frac{c}{m} \cdot \cos 2\alpha + y \cdot \sin 2\alpha - \frac{c}{m}$$

$$\rightarrow \quad x' = x \cdot cos\, 2\alpha + y \cdot \sin 2\alpha + \frac{c}{m} \cdot (\cos 2\alpha - 1) \text{ .... (3)}$$

$$\sin(2\alpha - \theta) = \frac{P'R'}{AP'} = \frac{y'}{AP'} \rightarrow y' = AP' \cdot \sin(2\alpha - \theta)$$

$$y' = AP' \cdot \sin 2\alpha \cos\theta - AP' \cdot \cos 2\alpha \sin\theta$$

$$y' = \left(x + \frac{c}{m}\right) \cdot \sin 2\alpha - y \cdot \cos 2\alpha$$

$$y' = x \cdot \sin 2\alpha - y \cdot \cos 2\alpha + \frac{c}{m} \cdot \sin 2\alpha \text{ .... (4)}$$

Dari (3) dan (4) diperoleh persamaan matriks

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \frac{c}{m} \begin{bmatrix} \cos 2\alpha - 1 \\ \sin 2\alpha \end{bmatrix}$$

Contoh:

1). Refleksi terhadap garis y = - x

m = - 1 maka tan $\alpha = -1 \rightarrow \alpha = 135^0$, $2\alpha = 270^0$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos 270^0 & \sin 270^0 \\ \sin 270^0 & -\cos 270^0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \frac{0}{-1} \begin{bmatrix} \cos 270^0 - 1 \\ \sin 270^0 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Dan matrik $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ disebut sebagai matrik transformasi yang bersesuaian dengan refleksi terhadap garis y = - x.

2). Refleksi terhadap garis y = x

m = 1 maka tan $\alpha = 1 \rightarrow \alpha = 45^0$, $2\alpha = 90^0$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos 90^0 & \sin 90^0 \\ \sin 90^0 & -\cos 90^0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \frac{0}{-1} \begin{bmatrix} \cos 90^0 - 1 \\ \sin 90^0 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Dan matrik $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ disebut sebagai matrik transformasi yang bersesuaian dengan refleksi terhadap garis y = x.