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SMAN 1 Bone-Bone, Luwu Utara, Sul-Sel

**Bagian terbaik dari seseorang adalah perbuatan-perbuatan baiknya dan kasihnya yang tidak diketahui orang lain (William Wordsworth)**

## [RUMUS CEPAT MATEMATIKA]

Limit Fungsi

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Materi ini dapat disebarluaskan secara bebas, untuk tujuan bukan komersial, dengan atau tanpa menyertakan sumber. Hak Cipta selamanya pada Allah Swt. Salam hangat selalu ...  
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1. SPMB 2002/Mat.Das/No.12

$$\lim_{x \rightarrow \infty} \frac{2x^2 - x + 4}{3x^2 - 5} = \dots$$

A.  $-\frac{5}{4}$

B.  $\frac{2}{3}$

C.  $\frac{3}{2}$

D.  $-\frac{5}{4}$

E.  $\sim$

**Smart**

**Remember**

☞ “ $\sim$ ” ucapkan BE >>SAR  
berarti : pilih koefisien  
variable pangkat be...sar

☞ Perhatikan Triknya ...

$$\lim_{x \rightarrow \sim} \frac{2x^2 - x + 4}{3x^2 - 5} = \frac{2}{3}$$

2. SPMB 2002/Mat.IPA/No.5

$$\lim_{x \rightarrow 0} \frac{\sin^2 3x \tan 2x - x^3}{x \tan^2 3x} = \dots$$

A.  $\frac{23}{9}$

B.  $\frac{19}{9}$

C.  $\frac{17}{9}$

D.  $\frac{8}{9}$

E. 0

**Smart**

**Remember**

$$\lim_{x \rightarrow 0} \frac{a \equiv^n bx}{p \equiv^n qx} = \frac{a \cdot b^n}{p \cdot q^n}$$

$\equiv$  di isi x, tg x atau sin x

$$\lim_{x \rightarrow 0} \frac{\sin^2 3x \tan 2x - x^3}{x \tan^2 3x} = \lim_{x \rightarrow 0} \frac{\sin^2 3x \tan 2x}{x \tan^2 3x} - \frac{x^3}{x \tan^2 3x} = \frac{3^2 \cdot 2}{3^2} - \frac{1}{3^2} = 2 - \frac{1}{9} = \frac{17}{9}$$

## 3. UMPTN '97

$$\lim_{x \rightarrow 0} \frac{(2x^3 + 3x)^3}{(5x^2 - 2x)(3x^2)} = \dots$$

- A.  $-1 \frac{1}{2}$
- B.  $-2 \frac{1}{2}$
- C.  $-3 \frac{1}{2}$
- D.  $-4 \frac{1}{2}$
- E.  $-5 \frac{1}{2}$

**Smart**

**Remember**

☞ “ $x \rightarrow 0$ ” ucapkan KE  
<<CIL  
berarti : pilih koefisien  
variable pangkat ke...cil

☞ Perhatikan Triknya :

$$\lim_{x \rightarrow 0} \frac{(2x^3 + 3x)^3}{(5x^2 - 2x)(3x^2)} = \frac{3^3}{-2 \cdot 3} = \frac{27}{-6} = -4 \frac{1}{2}$$

4.  $\lim_{x \rightarrow 1} \left( \frac{2}{x^2 - 1} - \frac{1}{x - 1} \right) = \dots$

- A.  $-\frac{3}{4}$
- B.  $-\frac{1}{2}$
- C.  $-\frac{1}{4}$
- D.  $\frac{1}{2}$
- E.  $\frac{3}{4}$

**Smart**

**Remember**

$$\begin{aligned} \frac{2}{x^2 - 1} - \frac{1}{x - 1} &= \frac{2}{(x-1)(x+1)} - \frac{1}{x-1} \\ &= \frac{2 - (x+1)}{(x-1)(x+1)} = \frac{-x+1}{x^2 - 1} \end{aligned}$$

☞ Bisa Anda Bayangkan  
Betapa mudahnya...

$$\lim_{x \rightarrow 1} \left( \frac{2}{x^2 - 1} - \frac{1}{x - 1} \right) = \lim_{x \rightarrow 1} \frac{-x+1}{x^2 - 1} = \frac{-1}{2 \cdot 1} = \frac{-1}{2} = -\frac{1}{2}$$

turunken

turunken (1)

6.  $\lim_{x \rightarrow 0} \frac{\tan 2x - 2 \tan x}{x^3} = \dots$

- A. 0
- B. 1
- C. 2
- D. 3
- E. 4

**Smart**

**Remember**

$\nabla \text{tg } 2ax - 2\text{tg } ax = 2a^3$

$\nabla$  Perhatikan, betapa mudahnya...

$\nabla \lim_{x \rightarrow 0} \frac{\tan 2x - 2 \tan x}{x^3} = \frac{2 \cdot 1^3}{1} = 2$

7.  $\lim_{x \rightarrow 3} \frac{\sqrt{x} - \sqrt{3}}{x - 3} = \dots$

A.  $\frac{1}{6}\sqrt{3}$

B.  $\frac{1}{3}\sqrt{3}$

C. 1

D.  $\oplus 3$

E. 3

**Smart**

**Remember**

$\lim_{x \rightarrow a} \frac{\sqrt{f(x)} - p}{g(x) - q} = \frac{f'(a)}{g'(a) \cdot 2p}$

$\lim_{x \rightarrow 3} \frac{\sqrt{x} - \sqrt{3}}{x - 3} = \frac{1}{1 \cdot 2\sqrt{3}} = \frac{1}{6}\sqrt{3}$

Mudeh...Khan...?

7.  $\lim_{x \rightarrow 7} \frac{x-7}{\sqrt{x}-\sqrt{7}} = \dots$

A.  $7 \oplus 7$

B.  $3 \oplus 7$

C.  $2 \oplus 7$

D.  $\frac{1}{2\sqrt{7}}$

E.  $\frac{1}{\sqrt{7}}$

**Smart**

**Remember**

$$\lim_{x \rightarrow a} \frac{f(x)-p}{\sqrt{g(x)-q}} =$$

$$\lim_{x \rightarrow 7} \frac{x-7}{\sqrt{x}-\sqrt{7}} = \frac{1.2\sqrt{7}}{1} = 2\sqrt{7}$$

Mudeh...Khan...?



## 9. UMPTN 1997

$$\lim_{x \rightarrow 0} \frac{2x^2 + x}{\sin x} = \dots$$

- A. 3
- B. 2
- C. 1
- D. 0
- E. -1

**Smart**

**Remember**

☞ “ $x \rightarrow 0$ ” ucapkan KE <<CIL  
berarti : pilih koefisien  
variable pangkat ke...cil

☞ Perhatikan Triknya :

$$\lim_{x \rightarrow 0} \frac{2x^2 + 1 \cdot x}{1 \cdot \sin x} = \frac{1}{1} = 1$$

## 10. UMPTN 1997

$$\lim_{x \rightarrow 0} \frac{\tan x}{x^2 + 2x} = \dots$$

- A. 2
- B. 1
- C. 0
- D.  $\frac{1}{2}$
- E.  $\frac{1}{4}$



### Smart

### Remember

☞ “ $x \rightarrow 0$ ” ucapkan KE <<CIL  
berarti : pilih koefisien  
variable pangkat ke...cil

☞ Perhatikan Triknya :

$$\lim_{x \rightarrow 0} \frac{1 \cdot \tan 1 \cdot x}{x^2 + 2x} = \frac{1 \cdot 1}{2} = \frac{1}{2}$$

12. Jika  $\lim_{x \rightarrow 0} \frac{1 - \cos ax}{x \tan x} = 8$ , maka nilai dari  $2a + 3 = \dots$

- A. 5
- B. 7
- C. 9
- D. 11
- E. 13

**Smart**

**Remember**

✎ Dalam limit :

$$1 - \cos ax = \frac{1}{2}a^2$$

✎  $\lim_{x \rightarrow 0} \frac{1 - \cos ax}{x \tan x} = 8$

$$\frac{1}{2}a^2$$

$$\frac{1}{2}a^2 = 8 \Rightarrow a^2 = 16. \text{ Jadi : } a = 4$$

✎ Maka  $2a + 3 = 8 + 3 = 11$

## 11. UMPTN 1998

Nilai  $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 2x}$  adalah...

- A. 0
- B. 2
- C. 4
- D. 6
- E. ~



### Smart

### Remember

☞  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)} \rightarrow$   
L'Hospital

☞  $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 2x} = \frac{3(2)^2}{2(2) - 2} = \frac{12}{2} = 6$

Mudah.....!?

## 12. UMPTN 1998

$$\lim_{x \rightarrow 2} \frac{\sin(x-2)}{x^2-4} = \dots$$

- A.  $-\frac{1}{4}$
- B.  $-\frac{1}{2}$
- C. 0
- D.  $\frac{1}{2}$
- E.  $\frac{1}{4}$

**Smart**

**Remember**

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)} \rightarrow$$

L'Hospital

$$\lim_{x \rightarrow 2} \frac{\sin(x-2)}{x^2-4} = \frac{\cos(2-2)}{2(2)} = \frac{1}{4}$$

Terlalu Mudah.....!?

## 13. UMPTN 1998

Nilai  $\lim_{x \rightarrow 0} \left( \frac{\tan 2x \cdot \tan 3x}{5x^2} \right)$  adalah...

A. 1

B.  $\frac{1}{5}$

C.  $\frac{2}{5}$

E.  $\frac{3}{5}$

D.  $\frac{6}{5}$

## Smart

## Remember

$$\lim_{x \rightarrow 0} \frac{\tan a \equiv \equiv}{b \equiv \equiv} = \frac{a}{b}$$

$\equiv \equiv$  di isi "variabel apa saja"

$$\lim_{x \rightarrow 0} \left( \frac{\tan 2x \cdot \tan 3x}{5x^2} \right) \frac{2 \cdot 3}{5} = \frac{6}{5}$$

Mudah Sekali.....

## 14. UMPTN 1999

$$\lim_{x \rightarrow 27} \frac{x-27}{\sqrt[3]{x}-3} = \dots$$

- A. 9
- B. 18
- C. 27
- D. 36
- E. 45

**Smart**

**Remember**

$$\lim_{x \rightarrow a} \frac{f(x) - p}{\sqrt[3]{g(x) - q}} = \frac{f'(a) \cdot 3q^2}{g'(a)}$$

$$\lim_{x \rightarrow 27} \frac{x-27}{\sqrt[3]{x}-3} = \frac{1 \cdot 3 \cdot 3^2}{1} = 27$$

## 15. UMPTN 1999

$$\lim_{x \rightarrow k} \frac{x - k}{\sin(x - k) + 2k - 2x} = \dots$$

- A. -1
- B. 0
- C.  $\frac{1}{3}$
- D.  $\frac{1}{2}$
- E. 1



### Smart

### Remember

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)} \rightarrow$$

L'Hospital

✎ Turunkan atas  
-bawah

$$\begin{aligned} \lim_{x \rightarrow k} \frac{x - k}{\sin(x - k) + 2k - 2x} &= \frac{1}{\cos(x - k) - 2} \\ &= \frac{1}{\cos 0 - 2} \\ &= \frac{1}{1 - 2} = -1 \end{aligned}$$



## 16. UMPTN 1999

$$\lim_{x \rightarrow 0} \frac{x(\cos^2 6x - 1)}{\sin 3x \cdot \tan^2 2x} = \dots$$

- A. 3
- B. -3
- C. 2
- D. -2
- E. -1



### Smart

### Remember

$$\lim_{x \rightarrow 0} \frac{\sin^n a}{\tan^n b} \equiv \frac{a^n}{b^n}$$

$\equiv$  di isi "variabel apa saja"

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{x(\cos^2 6x - 1)}{\sin 3x \cdot \tan^2 2x} &= \frac{x(-\sin^2 6x)}{\sin 3x \cdot \tan^2 2x} \\ &= \frac{-1 \cdot (6)^2}{3 \cdot (2)^2} = \frac{-36}{12} = -3 \end{aligned}$$

## 17. UMPTN 1999

Jika  $f(x) = x^2$  maka  $\lim_{x \rightarrow 3} \frac{f(x) - f(3)}{x - 3} = \dots$

- A. ~
- B. 0
- C. 3
- D. 6
- E. 9



### Smart

### Remember

- ✎  $f(x) = ax + b$ , maka :  
 $f(p) = ap + b$
- ✎  $f(x) = ax^2 + bx$ , maka :  
 $f(p) = ap^2 + bp$

✎ Perhatikan Triknya :

$$\begin{aligned}\lim_{x \rightarrow 3} \frac{f(x) - f(3)}{x - 3} &= \frac{x^2 - 9}{x - 3} = \frac{(x + 3)(x - 3)}{x - 3} \\ &= x + 3 \\ &= 3 + 3 \\ &= 6\end{aligned}$$

## 18. UMPTN 2000

$$\lim_{x \rightarrow 0} \frac{\cot x}{\cot 2x} = \dots$$

- A. 0
- B.  $\frac{1}{2}$
- C.  $\frac{1}{2} \oplus 2$
- D. 1
- E. 2



**Smart**

**Remember**

$$\lim_{x \rightarrow 0} \frac{\cot ax}{\cot bx} = \frac{b}{a}$$

📁  $\lim_{x \rightarrow 0} \frac{\cot x}{\cot 2x} = \frac{2}{1} = 2$

📁 Hanya membalik bil.yang menemani x  
Sangat Mudah bukan....?

19.  $\lim_{x \rightarrow 2} \frac{\sqrt{3x^2 + 8x - 3} - \sqrt{4x^2 + 9}}{x - 2} = \dots$

A.  $-\frac{4}{5}$

B. 0

C.  $\frac{2}{5}$

D.  $\frac{5}{2}$

E.  $\sim$

**Smart**

**Remember**

$$\lim_{x \rightarrow a} \frac{\sqrt{f(x)} - \sqrt{g(x)}}{h(x) - q} = \frac{f'(a) - g'(a)}{h'(a)2\sqrt{g(a)}}$$

☞ Perhatikan Triknya

$$\begin{aligned} \lim_{x \rightarrow 2} \frac{\sqrt{3x^2 + 8x - 3} - \sqrt{4x^2 + 9}}{x - 2} &= \frac{(6.2 + 8 - 8.2)}{1.2 \cdot \sqrt{4(2)^2 + 9}} \\ &= \frac{4}{2\sqrt{25}} = \frac{4}{10} = \frac{2}{5} \end{aligned}$$

20.  $\lim_{x \rightarrow 1} \frac{\sin(1 - \frac{1}{x}) \cos(1 - \frac{1}{x})}{x - 1} = \dots$

A. -1

B.  $-\frac{1}{2}$

C. 0

D.  $\frac{1}{2}$

E. 1

## Smart


## Remember



$\sin 2A = 2 \sin A \cos A$ ,

→ berarti :

$\sin A \cos A = \frac{1}{2} \sin 2A$

 
$$\lim_{x \rightarrow 1} \frac{\sin(1 - \frac{1}{x}) \cos(1 - \frac{1}{x})}{x - 1} = \frac{\sin 2(1 - \frac{1}{x})}{2(x - 1)}$$
$$= \frac{\frac{1}{x} \sin 2(1 - \frac{1}{x})}{\frac{1}{x} \cdot 2(x - 1)} = \frac{\frac{1}{x} \sin 2(1 - \frac{1}{x})}{2(1 - \frac{1}{x})} = \frac{1}{x} \cdot 1 = \frac{1}{1} = 1$$

21.  $\lim_{x \rightarrow \infty} (\sqrt{x(4x+5)} - \sqrt{4x^2-3}) = \dots$

- A.  $\sim$
- B. 8
- C.  $\frac{5}{4}$
- D.  $\frac{1}{2}$
- E. 0

**Smart**

**Remember**

$$\lim_{x \rightarrow \infty} (\sqrt{ax^2 + bx + c} - \sqrt{ax^2 + px + q}) = \frac{b-p}{2\sqrt{a}}$$

$\Rightarrow \lim_{x \rightarrow \infty} (\sqrt{x(4x+5)} - \sqrt{4x^2-3})$

$$\lim_{x \rightarrow \infty} (\sqrt{4x^2 + 5x} - \sqrt{4x^2 - 3}) = \frac{5-0}{2\sqrt{4}} = \frac{5}{4}$$

22. EBANAS 2002/No.17

$$\lim_{x \rightarrow \infty} 3x \sin \frac{1}{x} = \dots$$

- A.  $\sim$
- B. 0
- C. 1
- D. 2
- E. 3

**Smart**

**Remember**

✎ Missal :  $y = \frac{1}{x}$

$$x \rightarrow \sim \approx y \rightarrow 0$$

✎  $\lim_{x \rightarrow \infty} 3x \sin \frac{1}{x} \rightarrow \lim_{y \rightarrow 0} \frac{3}{y} \sin y = 3$

23. EBTANAS 2003/P-1/No.18

Nilai dari  $\lim_{x \rightarrow 9} \frac{x-9}{\sqrt{x}-3} = \dots$

- A. 6
- B. 4
- C. 3
- D. 1
- E. 0

**Smart**

**Remember**

- ✎ Akar di atas, tulis di "bawah"
- Akar di bawah, tulis di atas

pangkat akar      koefisien variabel

$$\lim_{x \rightarrow 9} \frac{1 \cdot x - 9}{\sqrt[2]{1 \cdot x - 3}} = \frac{1}{1} \left| \frac{2 \cdot 3}{1} = 6 \right.$$

pendamping akar





23. EBTANAS 2003/P-2/No.18

Nilai dari  $\lim_{x \rightarrow \infty} ((2x+1) - \sqrt{4x^2 - 3x + 6}) = \dots$

- B.  $\frac{4}{3}$
- B. 1
- C.  $\frac{7}{4}$
- D. 2
- E.  $\frac{5}{2}$



## Remember

$$\lim_{x \rightarrow \infty} \sqrt{ax^2 + bx + c} - \sqrt{ax^2 + px + q} = \frac{b-p}{2\sqrt{a}}$$

$$\lim_{x \rightarrow \infty} ((2x+1) - \sqrt{4x^2 - 3x + 6})$$

$$\lim_{x \rightarrow \infty} (\sqrt{(2x+1)^2} - \sqrt{4x^2 - 3x + 6})$$

$$\lim_{x \rightarrow \infty} \sqrt{4x^2 + 4x + 1} - \sqrt{4x^2 - 3x + 6} = \frac{4 - (-3)}{2\sqrt{4}} = \frac{7}{4}$$