

Lingkaran – Persamaan Lingkaran Melalui 3 Titik

Persamaan lingkaran melalui titik $A(x_1, y_1)$, $B(x_2, y_2)$, dan $C(x_3, y_3)$ adalah:

$$\text{determinan matrik} \begin{bmatrix} x^2 + y^2 & x & y & 1 \\ x_1^2 + y_1^2 & x_1 & y_1 & 1 \\ x_2^2 + y_2^2 & x_2 & y_2 & 1 \\ x_3^2 + y_3^2 & x_3 & y_3 & 1 \end{bmatrix} = 0 \quad \text{atau} \quad \begin{vmatrix} x^2 + y^2 & x & y & 1 \\ x_1^2 + y_1^2 & x_1 & y_1 & 1 \\ x_2^2 + y_2^2 & x_2 & y_2 & 1 \\ x_3^2 + y_3^2 & x_3 & y_3 & 1 \end{vmatrix} = 0$$

Cek:

Misalkan persamaan lingkaran adalah $x^2 + y^2 + Ax + By + C = 0$, maka:

$$x_1^2 + y_1^2 + Ax_1 + By_1 + C = 0 \quad \dots\dots\dots (1)$$

$$x_2^2 + y_2^2 + Ax_2 + By_2 + C = 0 \quad \dots\dots\dots (2)$$

$$x_3^2 + y_3^2 + Ax_3 + By_3 + C = 0 \quad \dots\dots\dots (3)$$

Kita eliminasi:

$$(1) - (2): x_1^2 - x_2^2 + y_1^2 - y_2^2 + A(x_1 - x_2) + B(y_1 - y_2) = 0 \quad \dots\dots\dots (4)$$

$$(1) - (3): x_1^2 - x_3^2 + y_1^2 - y_3^2 + A(x_1 - x_3) + B(y_1 - y_3) = 0 \quad \dots\dots\dots (5)$$

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Hak cipta dilindungi Allah, tak dilarangi menyebarkan sebagian atau seluruh isi tulisan ini dalam bentuk apapun selama ada manfaatnya, dan jangan lupa sisipkan DOA untuk kamu, Doa seorang muslim untuk saudaranya sesama muslim dari kejauhan tanpa diketahui olehnya akan Dikabulkan. Di atas kepalanya ada malaikat yg telah diutus, dan tiap kali ia berdoa untuk kebaikan, mk malaikat yg diutus tsb akan mengucapkan "Amin & kamu Juga akan mendapatkan seperti itu" (HR. Muslim 8/86).

$$(y_1 - y_3)(4): (x_1^2 - x_2^2 + y_1^2 - y_2^2)(y_1 - y_3) + A(x_1 - x_2)(y_1 - y_3) + B(y_1 - y_2)(y_1 - y_3) = 0$$

$$(y_1 - y_2)(5): (x_1^2 - x_3^2 + y_1^2 - y_3^2)(y_1 - y_2) + A(x_1 - x_3)(y_1 - y_2) + B(y_1 - y_2)(y_1 - y_3) = 0$$

$$(x_1^2 - x_2^2 + y_1^2 - y_2^2)(y_1 - y_3) - (x_1^2 - x_3^2 + y_1^2 - y_3^2)(y_1 - y_2) + A(x_1 - x_2)(y_1 - y_3) - A(x_1 - x_3)(y_1 - y_2) = 0$$

$$\Rightarrow A = \frac{(x_1^2 - x_3^2 + y_1^2 - y_3^2)(y_1 - y_2) - (x_1^2 - x_2^2 + y_1^2 - y_2^2)(y_1 - y_3)}{(x_1 - x_2)(y_1 - y_3) - (x_1 - x_3)(y_1 - y_2)}, \text{ atau}$$

$$\begin{aligned} \Rightarrow A &= \frac{(x_1^2 y_1 - x_3^2 y_1 + y_1^3 - y_1 y_3^2 - x_1^2 y_2 + x_3^2 y_2 - y_1^2 y_2 + y_2 y_3^2) - (x_1^2 y_1 - x_2^2 y_1 + y_1^3 - y_1 y_2^2 - x_1^2 y_3 + x_2^2 y_3 - y_1^2 y_3 + y_2^2 y_3)}{-x_1 y_3 - x_2 y_1 + x_2 y_3 + x_1 y_2 + x_3 y_1 - x_3 y_2} \\ &= \frac{(x_2^2 - x_3^2 + y_2^2 - y_3^2)y_1 + (x_3^2 - x_1^2 + y_3^2 - y_1^2)y_2 + (x_1^2 - x_2^2 + y_1^2 - y_2^2)y_3}{(x_3 - x_2)y_1 + (x_1 - x_3)y_2 + (x_2 - x_1)y_3} \end{aligned}$$

Substitusi A ke persamaan (4)

$$x_1^2 - x_2^2 + y_1^2 - y_2^2 + \frac{(x_1^2 - x_3^2 + y_1^2 - y_3^2)(x_1 - x_2)(y_1 - y_2) - (x_1^2 - x_2^2 + y_1^2 - y_2^2)(x_1 - x_2)(y_1 - y_3)}{(x_1 - x_2)(y_1 - y_3) - (x_1 - x_3)(y_1 - y_2)} + B(y_1 - y_2) = 0$$

$$\Rightarrow \frac{(x_1^2 - x_2^2 + y_1^2 - y_2^2)[(x_1 - x_2)(y_1 - y_3) - (x_1 - x_3)(y_1 - y_2)] + (x_1^2 - x_3^2 + y_1^2 - y_3^2)(x_1 - x_2)(y_1 - y_2) - (x_1^2 - x_2^2 + y_1^2 - y_2^2)(x_1 - x_2)(y_1 - y_3)}{(x_1 - x_2)(y_1 - y_3) - (x_1 - x_3)(y_1 - y_2)} + B(y_1 - y_2) = 0$$

$$\Rightarrow \frac{-(x_1^2 - x_2^2 + y_1^2 - y_2^2)(x_1 - x_3)(y_1 - y_2) + (x_1^2 - x_3^2 + y_1^2 - y_3^2)(x_1 - x_2)(y_1 - y_2)}{(x_1 - x_2)(y_1 - y_3) - (x_1 - x_3)(y_1 - y_2)} + B(y_1 - y_2) = 0$$

$$\Rightarrow B = \frac{(x_1^2 - x_2^2 + y_1^2 - y_2^2)(x_1 - x_3) - (x_1^2 - x_3^2 + y_1^2 - y_3^2)(x_1 - x_2)}{(x_1 - x_2)(y_1 - y_3) - (x_1 - x_3)(y_1 - y_2)}$$

atau

$$\begin{aligned}\Rightarrow B &= \frac{(x_1^3 - x_1x_2^2 + x_1y_1^2 - x_1y_2^2 - x_1^2x_3 + x_2^2x_3 - x_3y_1^2 + x_3y_2^2) - (x_1^3 - x_1x_3^2 + x_1y_1^2 - x_1y_3^2 - x_1^2x_2 + x_2x_3^2 - x_2y_1^2 + x_2y_3^2)}{-x_1y_3 - x_2y_1 + x_2y_3 + x_1y_2 + x_3y_1 - x_3y_2} \\ &= \frac{(x_3^2 - x_2^2 + y_3^2 - y_2^2)x_1 + (x_1^2 - x_3^2 + y_1^2 - y_3^2)x_2 + (x_2^2 - x_1^2 + y_2^2 - y_1^2)x_3}{(x_3 - x_2)y_1 + (x_1 - x_3)y_2 + (x_2 - x_1)y_3}\end{aligned}$$

Substitusi A dan B ke persamaan (1)

$$\begin{aligned}x_1^2 + y_1^2 + \frac{(x_2^2 - x_3^2 + y_2^2 - y_3^2)y_1 + (x_3^2 - x_1^2 + y_3^2 - y_1^2)y_2 + (x_1^2 - x_2^2 + y_1^2 - y_2^2)y_3}{(x_3 - x_2)y_1 + (x_1 - x_3)y_2 + (x_2 - x_1)y_3} x_1 \\ + \frac{(x_3^2 - x_2^2 + y_3^2 - y_2^2)x_1 + (x_1^2 - x_3^2 + y_1^2 - y_3^2)x_2 + (x_2^2 - x_1^2 + y_2^2 - y_1^2)x_3}{(x_3 - x_2)y_1 + (x_1 - x_3)y_2 + (x_2 - x_1)y_3} y_1 + C = 0 \\ \Rightarrow x_1^2 + y_1^2 + \frac{(x_2^2 - x_3^2 + y_2^2 - y_3^2)x_1y_1 + (x_3^2 - x_1^2 + y_3^2 - y_1^2)x_1y_2 + (x_1^2 - x_2^2 + y_1^2 - y_2^2)x_1y_3}{(x_3 - x_2)y_1 + (x_1 - x_3)y_2 + (x_2 - x_1)y_3} \\ + \frac{-(x_2^2 - x_3^2 + y_2^2 - y_3^2)x_1y_1 - (x_3^2 - x_1^2 + y_3^2 - y_1^2)x_2y_1 - (x_1^2 - x_2^2 + y_1^2 - y_2^2)x_3y_1}{(x_3 - x_2)y_1 + (x_1 - x_3)y_2 + (x_2 - x_1)y_3} + C = 0 \\ \Rightarrow x_1^2 + y_1^2 + \frac{(x_3^2 - x_1^2 + y_3^2 - y_1^2)(x_1y_2 - x_2y_1) + (x_1^2 - x_2^2 + y_1^2 - y_2^2)(x_1y_3 - x_3y_1)}{(x_3 - x_2)y_1 + (x_1 - x_3)y_2 + (x_2 - x_1)y_3} + C = 0\end{aligned}$$

$$\begin{aligned}
\Rightarrow C &= -x_1^2 - y_1^2 - \frac{(x_3^2 - x_1^2 + y_3^2 - y_1^2)(x_1y_2 - x_2y_1) + (x_1^2 - x_2^2 + y_1^2 - y_2^2)(x_1y_3 - x_3y_1)}{(x_3 - x_2)y_1 + (x_1 - x_3)y_2 + (x_2 - x_1)y_3} \\
&= -\frac{(x_1^2 + y_1^2)[(x_3 - x_2)y_1 + (x_1 - x_3)y_2 + (x_2 - x_1)y_3] - (x_3^2 - x_1^2 + y_3^2 - y_1^2)(x_1y_2 - x_2y_1) + (x_1^2 - x_2^2 + y_1^2 - y_2^2)(x_1y_3 - x_3y_1)}{(x_3 - x_2)y_1 + (x_1 - x_3)y_2 + (x_2 - x_1)y_3} \\
&= \frac{-(x_1^2 + y_1^2)x_3y_1 + (x_1^2 + y_1^2)x_2y_1 - (x_1^2 + y_1^2)x_1y_2 + (x_1^2 + y_1^2)x_3y_2 - (x_1^2 + y_1^2)x_2y_3 + (x_1^2 + y_1^2)x_1y_3}{(x_3 - x_2)y_1 + (x_1 - x_3)y_2 + (x_2 - x_1)y_3} \\
&\quad - \frac{-(x_1^2 + y_1^2)x_1y_2 + (x_1^2 + y_1^2)x_2y_1 + (x_3^2 + y_3^2)x_1y_2 - (x_3^2 + y_3^2)x_2y_1 + (x_1^2 + y_1^2)x_1y_3 - (x_1^2 + y_1^2)x_3y_1 - (x_2^2 + y_2^2)x_1y_3 + (x_2^2 + y_2^2)x_3y_1}{(x_3 - x_2)y_1 + (x_1 - x_3)y_2 + (x_2 - x_1)y_3} \\
&= \frac{(x_1^2 + y_1^2)x_3y_2 - (x_1^2 + y_1^2)x_2y_3 + (x_2^2 + y_2^2)x_1y_3 - (x_2^2 + y_2^2)x_3y_1 + (x_3^2 + y_3^2)x_2y_1 - (x_3^2 + y_3^2)x_1y_2}{(x_3 - x_2)y_1 + (x_1 - x_3)y_2 + (x_2 - x_1)y_3} \\
&= \frac{(x_1^2 + y_1^2)(x_3y_2 - x_2y_3) + (x_2^2 + y_2^2)(x_1y_3 - x_3y_1) + (x_3^2 + y_3^2)(x_2y_1 - x_1y_2)}{(x_3 - x_2)y_1 + (x_1 - x_3)y_2 + (x_2 - x_1)y_3}
\end{aligned}$$

Substitusi A, B, dan C ke persamaan lingkaran

$$x^2 + y^2 + Ax + By + C = 0$$

$$\begin{aligned}
x^2 + y^2 &+ \frac{(x_2^2 - x_3^2 + y_2^2 - y_3^2)y_1 + (x_3^2 - x_1^2 + y_3^2 - y_1^2)y_2 + (x_1^2 - x_2^2 + y_1^2 - y_2^2)y_3}{(x_3 - x_2)y_1 + (x_1 - x_3)y_2 + (x_2 - x_1)y_3} x \\
&+ \frac{(x_3^2 - x_2^2 + y_3^2 - y_2^2)x_1 + (x_1^2 - x_3^2 + y_1^2 - y_3^2)x_2 + (x_2^2 - x_1^2 + y_2^2 - y_1^2)x_3}{(x_3 - x_2)y_1 + (x_1 - x_3)y_2 + (x_2 - x_1)y_3} y \\
&+ \frac{(x_1^2 + y_1^2)(x_3y_2 - x_2y_3) + (x_2^2 + y_2^2)(x_1y_3 - x_3y_1) + (x_3^2 + y_3^2)(x_2y_1 - x_1y_2)}{(x_3 - x_2)y_1 + (x_1 - x_3)y_2 + (x_2 - x_1)y_3} = 0
\end{aligned}$$

$$(x^2 + y^2)[(x_3 - x_2)y_1 + (x_1 - x_3)y_2 + (x_2 - x_1)y_3] + x \cdot [((x_2^2 + y_2^2) - (x_3^2 + y_3^2))y_1 + ((x_3^2 + y_3^2) - (x_1^2 + y_1^2))y_2 + ((x_1^2 + y_1^2) - (x_2^2 + y_2^2))y_3] \\ + y \cdot [((x_3^2 + y_3^2) - (x_2^2 + y_2^2))x_1 + ((x_1^2 + y_1^2) - (x_3^2 + y_3^2))x_2 + ((x_2^2 + y_2^2) - (x_1^2 + y_1^2))x_3] \\ + [(x_1^2 + y_1^2)(x_3y_2 - x_2y_3) + (x_2^2 + y_2^2)(x_1y_3 - x_3y_1) + (x_3^2 + y_3^2)(x_2y_1 - x_1y_2)] = 0$$

$$(x^2 + y^2)[(x_3 - x_2)y_1 + (x_1 - x_3)y_2 + (x_2 - x_1)y_3] + x \cdot [(x_1^2 + y_1^2)(y_3 - y_2) + (x_2^2 + y_2^2)(y_1 - y_3) + (x_3^2 + y_3^2)(y_2 - y_1)] \\ + y \cdot [(x_1^2 + y_1^2)(x_2 - x_3) + (x_2^2 + y_2^2)(x_3 - x_1) + (x_3^2 + y_3^2)(x_1 - x_2)] + [(x_1^2 + y_1^2)(x_3y_2 - x_2y_3) + (x_2^2 + y_2^2)(x_1y_3 - x_3y_1) + (x_3^2 + y_3^2)(x_2y_1 - x_1y_2)] = 0$$

Kemudian, perhatikan determinan matrik yang disebutkan di awal

$$\begin{vmatrix} x^2 + y^2 & x & y & 1 \\ x_1^2 + y_1^2 & x_1 & y_1 & 1 \\ x_2^2 + y_2^2 & x_2 & y_2 & 1 \\ x_3^2 + y_3^2 & x_3 & y_3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow 0 = a_{11}c_{11} + a_{12}c_{12} + a_{13}c_{13} + a_{14}c_{14} \quad (\text{dipilih dari baris pertama})$$

$$\Rightarrow 0 = a_{11}(-1)^{1+1}M_{11} + a_{12}(-1)^{1+2}M_{12} + a_{13}(-1)^{1+3}M_{13} + a_{14}(-1)^{1+4}M_{14}$$

$$\Rightarrow 0 = (x^2 + y^2) \cdot \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} - x \cdot \begin{vmatrix} x_1^2 + y_1^2 & y_1 & 1 \\ x_2^2 + y_2^2 & y_2 & 1 \\ x_3^2 + y_3^2 & y_3 & 1 \end{vmatrix} + y \cdot \begin{vmatrix} x_1^2 + y_1^2 & x_1 & 1 \\ x_2^2 + y_2^2 & x_2 & 1 \\ x_3^2 + y_3^2 & x_3 & 1 \end{vmatrix} - 1 \cdot \begin{vmatrix} x_1^2 + y_1^2 & x_1 & y_1 \\ x_2^2 + y_2^2 & x_2 & y_2 \\ x_3^2 + y_3^2 & x_3 & y_3 \end{vmatrix}$$

M_{ij} adalah minor elemen a_{ij} pada matrik A yang didefinisikan sebagai determinan sub matrik A dengan menghapus baris ke i dan kolom ke j .

c_{ij} adalah kofaktor elemen a_{ij} pada matrik A didefinisikan dengan $c_{ij} = (-1)^{i+j}M_{ij}$.

$$0 = (x^2 + y^2)(x_1y_2 + x_2y_3 + x_3y_1 - x_1y_3 - x_2y_1 - x_3y_2) - x \cdot [(x_1^2 + y_1^2)(y_2 - y_3) + (x_2^2 + y_2^2)(y_3 - y_1) + (x_3^2 + y_3^2)(y_1 - y_2)] \\ + y \cdot [(x_1^2 + y_1^2)(x_2 - x_3) + (x_2^2 + y_2^2)(x_3 - x_1) + (x_3^2 + y_3^2)(x_1 - x_2)] - [(x_1^2 + y_1^2)(x_2y_3 - x_3y_2) + (x_2^2 + y_2^2)(x_3y_1 - x_1y_3) + (x_3^2 + y_3^2)(x_1y_2 - x_2y_1)]$$

$$0 = (x^2 + y^2)[(x_3 - x_2)y_1 + (x_1 - x_3)y_2 + (x_2 - x_1)y_3] - x \cdot [(x_1^2 + y_1^2)(y_2 - y_3) + (x_2^2 + y_2^2)(y_3 - y_1) + (x_3^2 + y_3^2)(y_1 - y_2)] \\ + y \cdot [(x_1^2 + y_1^2)(x_2 - x_3) + (x_2^2 + y_2^2)(x_3 - x_1) + (x_3^2 + y_3^2)(x_1 - x_2)] - [(x_1^2 + y_1^2)(x_2y_3 - x_3y_2) + (x_2^2 + y_2^2)(x_3y_1 - x_1y_3) + (x_3^2 + y_3^2)(x_1y_2 - x_2y_1)]$$

$$0 = (x^2 + y^2)[(x_3 - x_2)y_1 + (x_1 - x_3)y_2 + (x_2 - x_1)y_3] + x \cdot [(x_1^2 + y_1^2)(y_3 - y_2) + (x_2^2 + y_2^2)(y_1 - y_3) + (x_3^2 + y_3^2)(y_2 - y_1)] \\ + y \cdot [(x_1^2 + y_1^2)(x_2 - x_3) + (x_2^2 + y_2^2)(x_3 - x_1) + (x_3^2 + y_3^2)(x_1 - x_2)] + [(x_1^2 + y_1^2)(x_3y_2 - x_2y_3) + (x_2^2 + y_2^2)(x_1y_3 - x_3y_1) + (x_3^2 + y_3^2)(x_2y_1 - x_1y_2)]$$

Diperoleh persamaan yang sama, jadi persamaan lingkaran adalah benar. Selesai.

