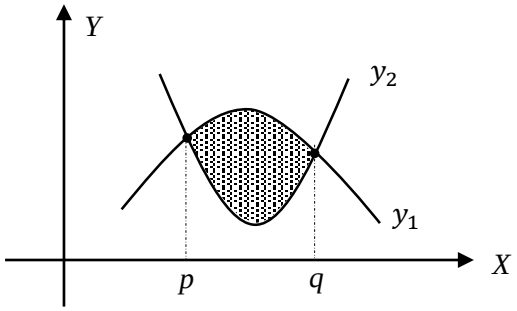
Integral – Luas Daerah Diantara 2 Kurva



Misalkan $y_1 - y_2 = ax^2 + bx + c$, maka luas daerah diantara kedua kurva adalah

$$L = \int_p^q (ax^2 + bx + c)dx = \left[\frac{1}{3}ax^3 + \frac{1}{2}bx^2 + cx\right]_p^q$$

$$= \left(\tfrac{1}{3}aq^3 + \tfrac{1}{2}bq^2 + cq\right) - \left(\tfrac{1}{3}ap^3 + \tfrac{1}{2}bp^2 + cp\right)$$

$$= \frac{2a(q^3 - p^3) + 3b(q^2 - p^2) + 6c(q - p)}{6}$$

$$= \frac{2a\big((q-p)^3 + 3qp(q-p)\big) + 3b\big((q-p)(q+p)\big) + 6c(q-p)}{6}$$

$$= \frac{2a\left(\left(\frac{\sqrt{D}}{a}\right)^3 + 3\frac{c}{a}\left(\frac{\sqrt{D}}{a}\right)\right) + 3b\left(\frac{\sqrt{D}}{a}\left(-\frac{b}{a}\right)\right) + 6c\left(\frac{\sqrt{D}}{a}\right)}{6}$$

$$= \frac{\frac{2D\sqrt{D}}{a^2} + \frac{6ac\sqrt{D}}{a^2} - \frac{3b^2\sqrt{D}}{a^2} + \frac{6ac\sqrt{D}}{a^2}}{6}$$

$$= \frac{2D\sqrt{D} - 3b^2\sqrt{D} + 12ac\sqrt{D}}{6a^2} \qquad = \frac{2D\sqrt{D} - 3\sqrt{D}(b^2 - 4ac)}{6a^2}$$

$$= \frac{2D\sqrt{D} - 3D\sqrt{D}}{6a^2} \qquad = \frac{-D\sqrt{D}}{6a^2}$$

karena luas daerah selalu positif, dapatlah kita tulis: $L = \left|\frac{D\sqrt{D}}{6a^2}\right|;\ D > 0$

Catatan:

1. p dan q adalah akar-akar dari $ax^2 + bx + c = 0$
2. $p, q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $p + q = -\frac{b}{a}, q - p = \frac{\sqrt{D}}{a}$, dan $pq = \frac{c}{a}$