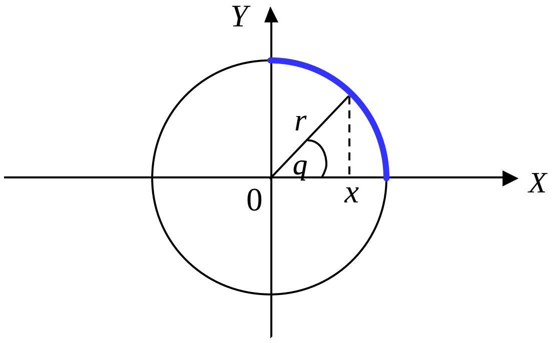
Integral – Keliling Lingkaran 2

Perhatikan gambar.

Diketahui persamaan suatu lingkaran $x^2 + y^2 = r^2$ atau $y = \sqrt{r^2 - x^2}$ untuk y positif.



Keliling lingkaran adalah 4 kali panjang kurva pada interval $[0,\ r]$ yaitu yang berwarna biru. Sehingga kita peroleh,

$$K = 4\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = 4\int_0^r \sqrt{1+\frac{x^2}{r^2-x^2}}\,dx = 4\int_0^r \sqrt{\frac{r^2}{r^2-x^2}}\,dx = 4r\int_0^r \sqrt{\frac{1}{r^2-x^2}}\,dx = 4r\int_0^r \frac{1}{\sqrt{r^2-x^2}}\,dx$$

Misalkan diketahui $x = r\sin q$ dan $dx = r\cos q\ dq$

Batas interval: $x = 0 \Rightarrow 0 = r\sin q \Rightarrow \sin q = 0 \Rightarrow q = 0$

$$x = r \Rightarrow r = r\sin q \Rightarrow \sin q = 1 \Rightarrow q = \frac{p}{2}$$

Maka keliling lingkaran menjadi,

$$K = 4r\int_0^r \frac{1}{\sqrt{r^2-x^2}}\,dx \quad = 4r\int_0^{p/2} \frac{r\cos q\ dq}{\sqrt{r^2 - r^2\sin^2 q}} \quad = 4r\int_0^{p/2} \frac{r\cos q\ dq}{\sqrt{r^2(1-\sin^2 q)}} \quad = 4r\int_0^{p/2} \frac{r\cos q\ dq}{\sqrt{r^2\cos^2 q}}$$

$$= 4r\int_0^{p/2} \frac{r\cos q\ dq}{r\cos q} \quad = 4r\int_0^{p/2} dq \quad = 4r[q]_0^{p/2} \quad = 4r\left(\frac{p}{2}\right) \quad = 2p\ r$$