

Integral – Integral Substitusi

Diberikan integral dalam bentuk $\int(ax+b)^n dx$

Misalkan $u = ax + b$, diperoleh

$$\frac{du}{dx} = a \Rightarrow du = a \cdot dx \Rightarrow dx = \frac{1}{a} du$$

Sehingga,

$$\begin{aligned}\int(ax+b)^n dx &= \int u^n \frac{1}{a} du \\ &= \frac{1}{a} \int u^n du \\ &= \frac{1}{a} \cdot \frac{1}{n+1} u^{n+1} + c \\ &= \frac{1}{a(n+1)} (ax+b)^{n+1} + c\end{aligned}$$

Contoh:

$$1. \int(4x+3)^5 dx = \int u^5 \frac{1}{4} du = \frac{1}{4} \int u^5 du = \frac{1}{4} \cdot \frac{1}{5+1} u^{5+1} + c = \frac{1}{4 \cdot 6} (4x+3)^6 + c = \frac{1}{24} (4x+3)^6 + c$$

$$u = 4x+3, du = 4dx \Rightarrow dx = \frac{1}{4} du$$

$$\begin{aligned}2. \int(x^4 - x^2)^3 (8x^3 - 4x) dx &= \int(x^4 - x^2)^3 2(4x^3 - 2x) dx \\ &= \int u^3 2(4x^3 - 2x) \frac{du}{(4x^3 - 2x)} \\ &= \int 2u^3 du \\ &= \frac{2}{4} u^4 + c \\ &= \frac{1}{2} (x^4 - x^2)^4 + c\end{aligned}$$

$$u = (x^4 - x^2), du = (4x^3 - 2x) dx \Rightarrow dx = \frac{du}{(4x^3 - 2x)}$$

Catatan:

Contoh no. 2 adalah soal yang lebih kompleks, namun langkah pengerjaan sama. Untuk u fungsi dalam x dan p konstanta,

$$\begin{aligned}\int u^n pu' dx &= p \int u^n u' \frac{1}{u'} du \\ &= p \int u^n du \\ &= p \cdot \frac{1}{n+1} u^{n+1} + c \\ &= \frac{p}{(n+1)} u^{n+1} + c\end{aligned}$$