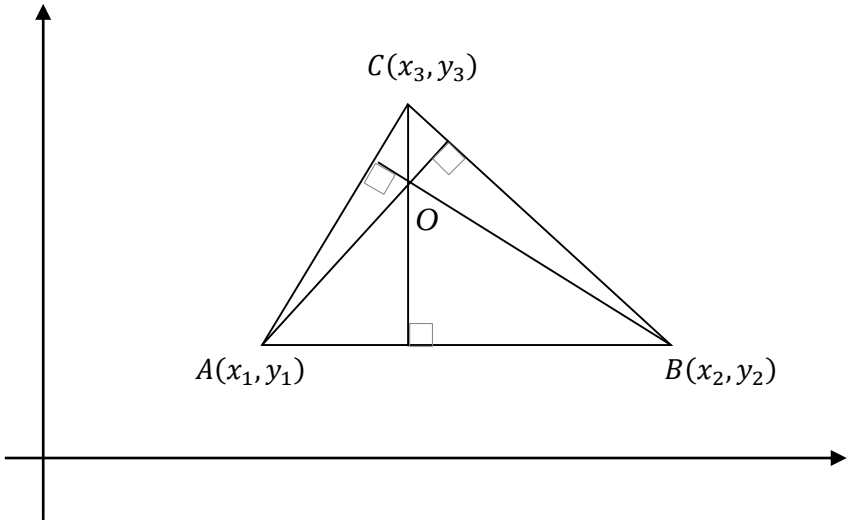
Geometri Koordinat - Koordinat Titik Tinggi Segitiga

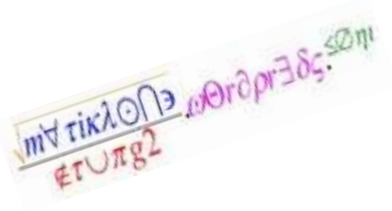


Perhatikan gambar!

Ketiga garis tinggi suatu segitiga berpotongan di 1 titik, disebut titik tinggi. Koordinat titik tinggi dapat kita cari dari titik perpotongan salah 2 garis tingginya. Dalam kesempatan ini, kita akan mencarinya menggunakan garis tinggi yang melalui titik sudut A, kita sebut garis *a* dan garis melalui titik sudut B, kita sebut garis *b*.

$m_{CB} = \frac{y_3 - y_2}{x_3 - x_2} \quad \rightarrow m_a = -\frac{x_3 - x_2}{y_3 - y_2}$ (garis CB dan garis *a* saling tegak lurus)

$m_{AC} = \frac{y_3 - y_1}{x_3 - x_1} \quad \rightarrow m_b = -\frac{x_3 - x_1}{y_3 - y_1}$ (garis AC dan garis *b* saling tegak lurus)



Garis $a \equiv y - y_1 = -\frac{x_3 - x_2}{y_3 - y_2}(x - x_1)$ $\rightarrow y(y_3 - y_2) - y_1 y_3 + y_1 y_2 = -x_3 x + x_2 x + x_1 x_3 - x_1 x_2$

$\rightarrow \quad y = \frac{y_1 y_3 - y_1 y_2 - x_3 x + x_2 x + x_1 x_3 - x_1 x_2}{y_3 - y_2}$

Garis $b \equiv y - y_2 = -\frac{x_3 - x_1}{y_3 - y_1}(x - x_2)$ $\rightarrow y(y_3 - y_1) - y_2 y_3 + y_1 y_2 = -x_3 x + x_1 x + x_2 x_3 - x_1 x_2$

$\rightarrow \quad y = \frac{y_2 y_3 - y_1 y_2 - x_3 x + x_1 x + x_2 x_3 - x_1 x_2}{y_3 - y_1}$

Titik tinggi adalah titik potong kedua garis, untuk mendapatkan absis titiknya, dapat kita samakan kedua persamaan di atas.

$$\frac{y_1 y_3 - y_1 y_2 - x_3 x + x_2 x + x_1 x_3 - x_1 x_2}{y_3 - y_2} = \frac{y_2 y_3 - y_1 y_2 - x_3 x + x_1 x + x_2 x_3 - x_1 x_2}{y_3 - y_1}$$

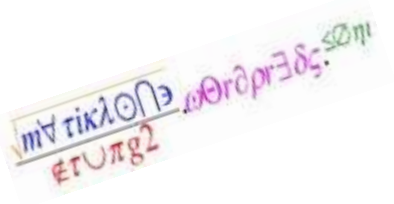
$$\rightarrow\ y_1{y_3}^2 - y_1y_2y_3 - x_3xy_3 + x_2xy_3 + x_1x_3y_3 - x_1x_2y_3 - {y_1}^2y_3 + {y_1}^2y_2 + x_3xy_1 - x_2xy_1 - x_1x_3y_1 + x_1x_2y_1$$
$$= y_2{y_3}^2 - y_1y_2y_3 - x_3xy_3 + x_1xy_3 + x_2x_3y_3 - x_1x_2y_3 - {y_2}^2y_3 + {y_2}^2y_1 + x_3xy_2 - x_1xy_2 - x_2x_3y_2 + x_1x_2y_2$$

$$\rightarrow\ y_1{y_3}^2 + x_2xy_3 + x_1x_3y_3 - {y_1}^2y_3 + {y_1}^2y_2 + x_3xy_1 - x_2xy_1 - x_1x_3y_1 + x_1x_2y_1$$
$$= y_2{y_3}^2 + x_1xy_3 + x_2x_3y_3 - {y_2}^2y_3 + {y_2}^2y_1 + x_3xy_2 - x_1xy_2 - x_2x_3y_2 + x_1x_2y_2$$

$$\rightarrow\ x(x_2y_3 + x_3y_1 + x_1y_2 - x_2y_1 - x_1y_3 - x_3y_2) = x_1x_3y_1 + y_1{y_2}^2 + x_1x_2y_2 + y_2{y_3}^2 + x_2x_3y_3 + {y_1}^2y_3 - x_2x_3y_2 - {y_1}^2y_2$$
$$- x_1x_3y_3 - {y_2}^2y_3 - x_1x_2y_1 - y_1{y_3}^2$$

$$\rightarrow\ x = \frac{x_1x_3y_1 + y_1{y_2}^2 + x_1x_2y_2 + y_2{y_3}^2 + x_2x_3y_3 + {y_1}^2y_3 - x_2x_3y_2 - {y_1}^2y_2 - x_1x_3y_3 - {y_2}^2y_3 - x_1x_2y_1 - y_1{y_3}^2}{x_2y_3 + x_3y_1 + x_1y_2 - x_2y_1 - x_1y_3 - x_3y_2}$$

$$= \frac{\begin{vmatrix} y_1 & x_2x_3 + {y_1}^2 & 1 \\ y_2 & x_1x_3 + {y_2}^2 & 1 \\ y_3 & x_1x_2 + {y_3}^2 & 1 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}}$$



Dengan cara yang sama (silakan Anda cari untuk latihan), dengan menyatakan persamaan garis dalam *x* = b*y* + c, akan kita dapatkan:

$$y = \frac{\begin{vmatrix} {x_1}^2 + y_2 y_3 & x_1 & 1 \\ {x_2}^2 + y_1 y_3 & x_2 & 1 \\ {x_3}^2 + y_1 y_2 & x_3 & 1 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}}$$

Jadi, koordinat titik tinggi segitiga ABC adalah:

$$O(x,y) = O\left(\frac{\begin{vmatrix} y_1 & x_2 x_3 + {y_1}^2 & 1 \\ y_2 & x_1 x_3 + {y_2}^2 & 1 \\ y_3 & x_1 x_2 + {y_3}^2 & 1 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}}, \quad \frac{\begin{vmatrix} {x_1}^2 + y_2 y_3 & x_1 & 1 \\ {x_2}^2 + y_1 y_3 & x_2 & 1 \\ {x_3}^2 + y_1 y_2 & x_3 & 1 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}} \right)$$