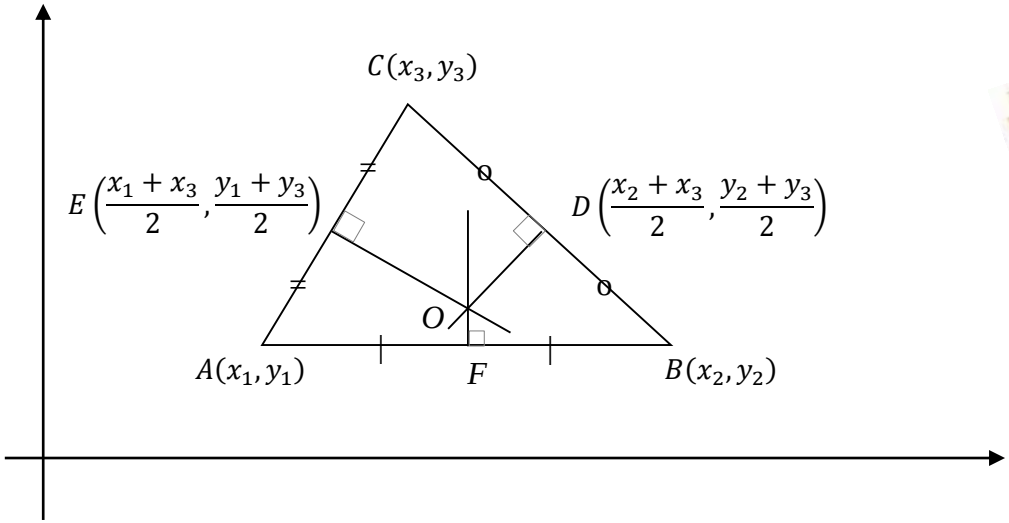
Geometri Koordinat - Koordinat Titik Sumbu Segitiga



Perhatikan gambar!

Ketiga garis sumbu suatu segitiga berpotongan di 1 titik, disebut titik sumbu. Koordinat titik sumbu dapat kita cari dari titik perpotongan salah 2 garis sumbunya. Kita akan mencarinya menggunakan garis sumbu yang melalui titik D, kita sebut garis *a* dan garis melalui titik E, kita sebut dengan garis *e*.

$m_{BC} = \frac{y_3 - y_2}{x_3 - x_2} \rightarrow m_d = -\frac{x_3 - x_2}{y_3 - y_2}$ (garis BC dan garis *d* saling tegak lurus)

$m_{AC} = \frac{y_3 - y_1}{x_3 - x_1} \rightarrow m_e = -\frac{x_3 - x_1}{y_3 - y_1}$ (garis AC dan garis *e* saling tegak lurus)

Garis $d \equiv y - \frac{y_2 + y_3}{2} = -\frac{x_3 - x_2}{y_3 - y_2}\left(x - \frac{x_2 + x_3}{2}\right) \rightarrow 2y - \left(y_2 + y_3\right) = \frac{x_2 - x_3}{y_3 - y_2}\left(2x - \left(x_2 + x_3\right)\right)$

$$\rightarrow 2y(y_3 - y_2) - y_2 y_3 - {y_3}^2 + {y_2}^2 + y_2 y_3 = 2x_2 x - 2x_3 x - {x_2}^2 + x_2 x_3 - x_2 x_3 + {x_3}^2$$

$$\rightarrow \quad 2y(y_3 - y_2) - {y_3}^2 + {y_2}^2 = 2x_2 x - 2x_3 x - {x_2}^2 + {x_3}^2$$

$$\rightarrow \quad 2y = \frac{{y_3}^2 - {y_2}^2 + 2x_2 x - 2x_3 x - {x_2}^2 + {x_3}^2}{y_3 - y_2}$$

Garis $e \equiv y - \frac{y_1 + y_3}{2} = -\frac{x_3 - x_1}{y_3 - y_1}\left(x - \frac{x_1 + x_3}{2}\right) \rightarrow 2y - \left(y_1 + y_3\right) = \frac{x_1 - x_3}{y_3 - y_1}\left(2x - \left(x_1 + x_3\right)\right)$

$$\rightarrow 2y(y_3 - y_1) - y_1 y_3 - {y_3}^2 + {y_1}^2 + y_1 y_3 = 2x_1 x - 2x_3 x - {x_1}^2 + x_1 x_3 - x_1 x_3 + {x_3}^2$$

$$\rightarrow \quad 2y(y_3 - y_1) - {y_3}^2 + {y_1}^2 = 2x_1 x - 2x_3 x - {x_1}^2 + {x_3}^2$$

$$\rightarrow \quad 2y = \frac{{y_3}^2 - {y_1}^2 + 2x_1 x - 2x_3 x - {x_1}^2 + {x_3}^2}{y_3 - y_1}$$

Titik tinggi adalah titik potong kedua garis, untuk mendapatkan absis titiknya, dapat kita samakan kedua persamaan di atas.

$$\frac{{y_3}^2 - {y_2}^2 + 2x_2x - 2x_3x - {x_2}^2 + {x_3}^2}{y_3 - y_2} = \frac{{y_3}^2 - {y_1}^2 + 2x_1x - 2x_3x - {x_1}^2 + {x_3}^2}{y_3 - y_1}$$

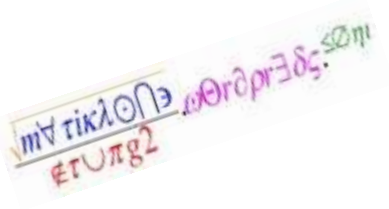
$$\rightarrow \; {y_3}^3 - {y_2}^2y_3 + 2x_2xy_3 - 2x_3xy_3 - {x_2}^2y_3 + {x_3}^2y_3 - y_1{y_3}^2 + y_1{y_2}^2 - 2x_2xy_1 + 2x_3xy_1 + {x_2}^2y_1 - {x_3}^2y_1$$
$$= {y_3}^3 - {y_1}^2y_3 + 2x_1xy_3 - 2x_3xy_3 - {x_1}^2y_3 + {x_3}^2y_3 - y_2{y_3}^2 + y_2{y_1}^2 - 2x_1xy_2 + 2x_3xy_2 + {x_1}^2y_2 - {x_3}^2y_2$$

$$\rightarrow \; -{y_2}^2y_3 + 2x_2xy_3 - {x_2}^2y_3 - y_1{y_3}^2 + y_1{y_2}^2 - 2x_2xy_1 + 2x_3xy_1 + {x_2}^2y_1 - {x_3}^2y_1$$
$$= -{y_1}^2y_3 + 2x_1xy_3 - {x_1}^2y_3 - y_2{y_3}^2 + y_2{y_1}^2 - 2x_1xy_2 + 2x_3xy_2 + {x_1}^2y_2 - {x_3}^2y_2$$

$$\rightarrow \quad 2x(x_1y_2 + x_2y_3 + x_3y_1 - x_2y_1 - x_3y_2 - x_1y_3) = \; {x_1}^2y_2 + y_2{y_1}^2 + {x_2}^2y_3 + {y_2}^2y_3 + {x_3}^2y_1 + y_1{y_3}^2 - {x_2}^2y_1$$
$$- y_1{y_2}^2 - {x_3}^2y_2 - y_2{y_3}^2 - {x_1}^2y_3 - {y_1}^2y_3$$

$$\rightarrow \quad x = \frac{{x_1}^2y_2 + y_2{y_1}^2 + {x_2}^2y_3 + {y_2}^2y_3 + {x_3}^2y_1 + y_1{y_3}^2 - {x_2}^2y_1 - y_1{y_2}^2 - {x_3}^2y_2 - y_2{y_3}^2 - {x_1}^2y_3 - {y_1}^2y_3}{2(x_1y_2 + x_2y_3 + x_3y_1 - x_2y_1 - x_3y_2 - x_1y_3)}$$

$$= \frac{\begin{vmatrix} {x_1}^2 + {y_1}^2 & y_1 & 1 \\ {x_2}^2 + {y_2}^2 & y_2 & 1 \\ {x_3}^2 + {y_3}^2 & y_3 & 1 \end{vmatrix}}{2\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}}$$



Dengan cara yang sama (silakan dicari untuk latihan), dengan menyatakan persamaan garis dalam $x = by + c$, akan kita dapatkan:

$$y = \frac{\begin{vmatrix} x_1 & {x_1}^2 + {y_1}^2 & 1 \\ x_2 & {x_2}^2 + {y_2}^2 & 1 \\ x_3 & {x_3}^2 + {y_3}^2 & 1 \end{vmatrix}}{2\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}}$$

Jadi, koordinat titik subu segitiga ABC adalah:

$$O(x,y) = O\left(\frac{\begin{vmatrix} {x_1}^2 + {y_1}^2 & y_1 & 1 \\ {x_2}^2 + {y_2}^2 & y_2 & 1 \\ {x_3}^2 + {y_3}^2 & y_3 & 1 \end{vmatrix}}{2\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}}, \quad \frac{\begin{vmatrix} x_1 & {x_1}^2 + {y_1}^2 & 1 \\ x_2 & {x_2}^2 + {y_2}^2 & 1 \\ x_3 & {x_3}^2 + {y_3}^2 & 1 \end{vmatrix}}{2\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}} \right)$$