

## Barisan dan Deret – Bader Bertingkat 2

Misalkan deret bertingkat  $x_1 + x_2 + x_3 + x_4 + \dots + x_n$  dimana  $y_k = x_{k+1} - x_k$  dan  $z = y_{k+1} - y_k$ .

$$\begin{array}{ccccccc}
 x_1 & + & x_2 & + & x_3 & + & x_4 & + & \dots & + & x_n \\
 \underbrace{\phantom{x_1}} & & \underbrace{\phantom{x_2}} & & \underbrace{\phantom{x_3}} & & \underbrace{\phantom{x_4}} & & & & \\
 y_1 & & y_2 & & y_3 & & & & & & \\
 \underbrace{\phantom{y_1}} & & \underbrace{\phantom{y_2}} & & & & & & & & \\
 z & & z & & & & & & & & 
 \end{array}$$

$$U_n = an^2 + bn + c$$

$$\left. \begin{array}{l}
 \Rightarrow x_1 = a + b + c \quad \dots(1) \\
 x_2 = 4a + 2b + c \quad \dots(2) \\
 x_3 = 9a + 3b + c \quad \dots(3)
 \end{array} \right\} \begin{array}{l}
 (2) - (1): y_1 = 3a + b \quad \dots(4) \\
 (3) - (2): y_2 = 5a + b \quad \dots(5)
 \end{array} \left. \right\} (5) - (4): z = 2a \Rightarrow a = \frac{z}{2}$$

$$\otimes y_1 = 3a + b \quad \Rightarrow b = y_1 - 3a \quad \Rightarrow b = y_1 - \frac{3z}{2}$$

$$\otimes x_1 = a + b + c \Rightarrow c = x_1 - b - a \Rightarrow c = x_1 - \left(y_1 - \frac{3z}{2}\right) - \frac{z}{2} \Rightarrow c = x_1 - y_1 + z$$

$$\begin{aligned}
 U_n &= an^2 + bn + c \\
 &= \frac{z}{2}n^2 + \left(y_1 - \frac{3z}{2}\right)n + (x_1 - y_1 + z)
 \end{aligned}$$

Contoh soal:

$$\begin{array}{ccccccc}
 6 & + & 11 & + & 18 & + & 27 & + & \dots & + & 123 \\
 \underbrace{\phantom{6}} & & \underbrace{\phantom{11}} & & \underbrace{\phantom{18}} & & \underbrace{\phantom{27}} & & & & \\
 5 & & 7 & & 9 & & & & & & \\
 \underbrace{\phantom{5}} & & \underbrace{\phantom{7}} & & & & & & & & \\
 2 & & 2 & & & & & & & & 
 \end{array}$$

$$\left\langle \begin{aligned}
 U_n &= \frac{2}{2}n^2 + \left(5 - \frac{3 \cdot 2}{2}\right)n + (6 - 5 + 2) \\
 &= n^2 + 2n + 3
 \end{aligned} \right.$$

Bab Terkait: Notasi Sigma, Induksi Matematika.