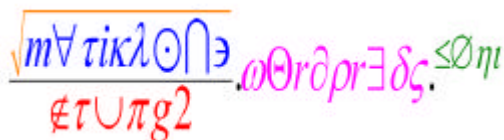


## Galeri Soal



65 Soal dengan Pembahasan, 315 Soal Latihan

Dirangkum Oleh:  
**Anang Wibowo, S.Pd**



April 2012

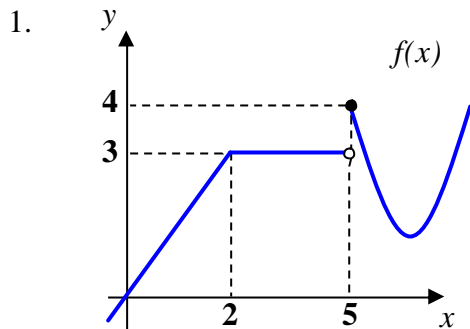
MatikZone's Series

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# Soal-soal Limit dan Penyelesaiannya



Dari gambar di samping, tentukan:

- a).  $\lim_{x \rightarrow 2^-} f(x)$ ,  $\lim_{x \rightarrow 2^+} f(x)$  dan  $\lim_{x \rightarrow 2} f(x)$  jika ada.
- b).  $\lim_{x \rightarrow 5^-} f(x)$ ,  $\lim_{x \rightarrow 5^+} f(x)$ , dan  $\lim_{x \rightarrow 5} f(x)$  jika ada.

**Jawab:**

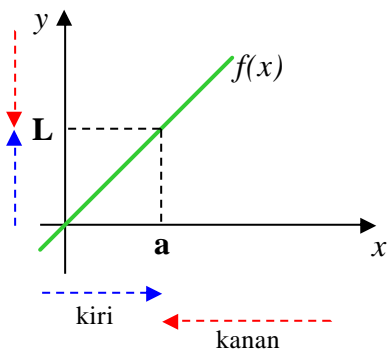
## Limit kanan dan limit kiri

\*)  $\lim_{x \rightarrow a^+} f(x) = L$ , artinya bilamana  $x$  mendekati  $a$  dari kanan, maka nilai  $f(x)$  mendekati  $L$ .

\*)  $\lim_{x \rightarrow a^-} f(x) = L$ , artinya bilamana  $x$  mendekati  $a$  dari kiri, maka nilai  $f(x)$  mendekati  $L$ .

## Definisi limit

$$\lim_{x \rightarrow a} f(x) = L \text{ (ada)} \Leftrightarrow \lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a^-} f(x) = L$$



Dari soal di atas dapat ditentukan bahwa:

- a).  $\lim_{x \rightarrow 2^-} f(x) = 3$  dan  $\lim_{x \rightarrow 2^+} f(x) = 3$  maka  $\lim_{x \rightarrow 2} f(x) = 3$
- b).  $\lim_{x \rightarrow 5^-} f(x) = 3$  dan  $\lim_{x \rightarrow 5^+} f(x) = 4$ , limit kiri dan limit kanan tidak sama maka  $\lim_{x \rightarrow 5} f(x)$  Tidak Ada

2. Jika diketahui  $f(x) = \begin{cases} 4x - 1; & \text{jk } x < 2 \\ x^2 + 3; & \text{jk } x \geq 2 \end{cases}$  maka tentukan nilai dari  $\lim_{x \rightarrow 2^-} f(x)$ ,  $\lim_{x \rightarrow 2^+} f(x)$ , dan  $\lim_{x \rightarrow 2} f(x)$

**Jawab:**

- $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} 4x - 1 = 4 \cdot 2 - 1 = 8 - 1 = 7$  (limit kiri, dari kiri, digunakan fungsi pertama)
- $\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} x^2 + 3 = 2^2 + 3 = 4 + 3 = 7$  (limit kanan, dari kanan, digunakan fungsi kedua)
- $\lim_{x \rightarrow 2} f(x) = 7$  (limit kiri = limit kanan)

3. Tentukan nilai limit dari:

a). $\lim_{x \rightarrow 9} 788$	c). $\lim_{x \rightarrow 3} (5x - 6)$	e). $\lim_{x \rightarrow 2} \frac{x - 2}{x + 2}$
b). $\lim_{x \rightarrow 8} 7x$	d). $\lim_{x \rightarrow -3} \frac{5x - 6}{x + 1}$	f). $\lim_{x \rightarrow -4} \frac{8 - x}{x + 4}$

**Jawab:**

Untuk  $\lim_{x \rightarrow a} f(x)$  diselesaikan dengan cara **substitusi (langkah ini tidak boleh ditinggalkan)**

- Jika  $f(a) = c$  maka  $\lim_{x \rightarrow a} f(x) = c$
- Jika  $f(a) = \frac{c}{0}$  maka  $\lim_{x \rightarrow a} f(x)$  Tidak Ada, Tak Hingga, atau Min Tak Hingga (cek grafik)
- Jika  $f(a) = \frac{0}{c}$  maka  $\lim_{x \rightarrow a} f(x) = 0$
- Jika  $f(a) = \frac{0}{0}$  maka dilakukan **faktorisasi** atau **perkalian dengan sekawan**

**Sehingga:**

a).  $\lim_{x \rightarrow 9} 788 = 788$

b).  $\lim_{x \rightarrow 8} 7x = 7 \cdot 8 = 56$

c).  $\lim_{x \rightarrow 3} (5x - 6) = 5 \cdot 3 - 6 = 15 - 6 = 9$

d).  $\lim_{x \rightarrow -3} \frac{5x - 6}{x + 1} = \frac{5(-3) - 6}{-3 + 1} = \frac{-15 - 6}{-2} = \frac{-21}{-2} = \frac{21}{2}$

e).  $\lim_{x \rightarrow 2} \frac{x - 2}{x + 2} = \frac{2 - 2}{2 + 2} = \frac{0}{4} = 0$

f).  $\lim_{x \rightarrow -4} \frac{8 - x}{x + 4} = \frac{8 - (-4)}{-4 + 4} = \frac{12}{0}$  Tidak ada (berdasar grafik)

#### 4. Penyelesaian dengan faktorisasi

a).  $\lim_{x \rightarrow 2} \frac{x-2}{x^2-5x+6} = \frac{2-2}{2^2-5 \cdot 2+6} = \frac{0}{0}$  BTT, maka

$$\lim_{x \rightarrow 2} \frac{x-2}{x^2-5x+6} = \lim_{x \rightarrow 2} \frac{x-2}{(x-2)(x-3)} = \lim_{x \rightarrow 2} \frac{1}{x-3} = \frac{1}{2-3} = \frac{1}{-1} = -1$$

b).  $\lim_{x \rightarrow -1} \frac{x^2+3x+2}{x^2-5x-6} = \frac{(-1)^2+3(-1)+2}{(-1)^2-5(-1)-6} = \frac{1-3+2}{1+5-6} = \frac{0}{0}$  BTT, maka

$$\lim_{x \rightarrow -1} \frac{x^2+3x+2}{x^2-5x-6} = \lim_{x \rightarrow -1} \frac{(x+1)(x+2)}{(x+1)(x-6)} = \lim_{x \rightarrow -1} \frac{(x+2)}{(x-6)} = \frac{-1+2}{-1-6} = \frac{1}{-7} = -\frac{1}{7}$$

c).  $\lim_{x \rightarrow 0} \frac{x^3-5x^2+3x}{2x-7x^2} = \frac{0^3-5 \cdot 0^2+3 \cdot 0}{2 \cdot 0-7 \cdot 0^2} = \frac{0}{0}$  BTT, maka

$$\lim_{x \rightarrow 0} \frac{x^3-5x^2+3x}{2x-7x^2} = \lim_{x \rightarrow 0} \frac{x(x^2-5x+3)}{x(2-7x)} = \lim_{x \rightarrow 0} \frac{(x^2-5x+3)}{(2-7x)} = \frac{0-5 \cdot 0+3}{2-7 \cdot 0} = \frac{3}{2}$$

d).  $\lim_{x \rightarrow 2} \frac{x^3+x^2-8x+4}{x^3-2x^2-x+2} = \lim_{x \rightarrow 2} \frac{(x-2)(x^2+3x-2)}{(x-2)(x^2-1)} = \lim_{x \rightarrow 2} \frac{x^2+3x-2}{x^2-1} = \frac{2^2+3 \cdot 2-2}{2^2-1} = \frac{8}{3}$

e).  $\lim_{x \rightarrow 4} \frac{4-x}{x^3-64} = \lim_{x \rightarrow 4} \frac{4-x}{(x-4)(x^2+4x+16)} = \lim_{x \rightarrow 4} \frac{-(x-4)}{(x-4)(x^2+4x+16)}$

$$= \lim_{x \rightarrow 4} -\frac{1}{x^2+4x+16} = -\frac{1}{4^2+4 \cdot 4+16} = -\frac{1}{48}$$

f).  $\lim_{x \rightarrow \frac{3}{2}} \sqrt{\frac{8x^3-27}{4x^2-9}} = \lim_{x \rightarrow \frac{3}{2}} \sqrt{\frac{(2x)^3-3^3}{(2x)^2-3^2}} = \lim_{x \rightarrow \frac{3}{2}} \sqrt{\frac{(2x-3)(4x^2+6x+9)}{(2x-3)(2x+3)}} = \lim_{x \rightarrow \frac{3}{2}} \sqrt{\frac{4x^2+6x+9}{2x+3}}$

$$= \sqrt{\frac{4 \cdot \left(\frac{3}{2}\right)^2 + 6 \cdot \left(\frac{3}{2}\right) + 9}{2 \cdot \left(\frac{3}{2}\right) + 3}} = \sqrt{\frac{9+9+9}{3+3}} = \sqrt{\frac{27}{6}} = \sqrt{\frac{9}{2}} = 3\sqrt{\frac{1}{2}}$$

#### 5. Penyelesaian dengan perkalian bentuk sekawan (merasionalkan bentuk akar)

a).  $\lim_{x \rightarrow 2} \frac{3-\sqrt{4x+1}}{x-2} = \frac{3-\sqrt{8+1}}{2-2} = \frac{0}{0}$  BTT, maka

$$\lim_{x \rightarrow 2} \frac{3-\sqrt{4x+1}}{x-2} = \lim_{x \rightarrow 2} \frac{3-\sqrt{4x+1}}{x-2} \cdot \frac{3+\sqrt{4x+1}}{3+\sqrt{4x+1}} = \lim_{x \rightarrow 2} \frac{9-(4x+1)}{(x-2)(3+\sqrt{4x+1})}$$

$$= \lim_{x \rightarrow 2} \frac{8-4x}{(x-2)(3+\sqrt{4x+1})} = \lim_{x \rightarrow 2} \frac{-4(x-2)}{(x-2)(3+\sqrt{4x+1})}$$

$$= \lim_{x \rightarrow 2} \frac{-4}{3+\sqrt{4x+1}} = \frac{-4}{3+\sqrt{4 \cdot 2+1}} = \frac{-4}{3+3} = \frac{-4}{6} = -\frac{2}{3}$$

b).  $\lim_{x \rightarrow 3} \frac{\sqrt{x+2} - \sqrt{2x-1}}{\sqrt{2x-3} - \sqrt{x}} = \frac{0}{0}$  BTT, maka

$$\lim_{x \rightarrow 3} \frac{\sqrt{x+2} - \sqrt{2x-1}}{\sqrt{2x-3} - \sqrt{x}} = \lim_{x \rightarrow 3} \frac{\sqrt{x+2} - \sqrt{2x-1}}{\sqrt{2x-3} - \sqrt{x}} \cdot \frac{\sqrt{x+2} + \sqrt{2x-1}}{\sqrt{x+2} + \sqrt{2x-1}}$$

Dikali sekawan pembilang

$$= \lim_{x \rightarrow 3} \frac{(x+2) - (2x-1)}{(\sqrt{2x-3} - \sqrt{x})(\sqrt{x+2} + \sqrt{2x-1})}$$

Jika disubstitusi, masih didapat 0/0

$$= \lim_{x \rightarrow 3} \frac{-x+3}{(\sqrt{2x-3} - \sqrt{x})(\sqrt{x+2} + \sqrt{2x-1})}$$

$$= \lim_{x \rightarrow 3} \frac{-x+3}{(\sqrt{2x-3} - \sqrt{x})(\sqrt{x+2} + \sqrt{2x-1})} \cdot \frac{(\sqrt{2x-3} + \sqrt{x})}{(\sqrt{2x-3} + \sqrt{x})}$$

Dikali sekawan penyebut

$$= \lim_{x \rightarrow 3} \frac{(-x+3)(\sqrt{2x-3} + \sqrt{x})}{(\sqrt{x+2} + \sqrt{2x-1})((2x-3) - (x))}$$

$$= \lim_{x \rightarrow 3} \frac{-(x-3)(\sqrt{2x-3} + \sqrt{x})}{(\sqrt{x+2} + \sqrt{2x-1})(x-3)}$$

$$= \lim_{x \rightarrow 3} \frac{-(\sqrt{2x-3} + \sqrt{x})}{(\sqrt{x+2} + \sqrt{2x-1})}$$

$$= \frac{-(\sqrt{2 \cdot 3 - 3} + \sqrt{3})}{(\sqrt{3+2} + \sqrt{2 \cdot 3 - 1})} = \frac{-(\sqrt{3} + \sqrt{3})}{\sqrt{5} + \sqrt{5}} = -\frac{2\sqrt{3}}{2\sqrt{5}} = -\frac{\sqrt{3}}{\sqrt{5}}$$

c).  $\lim_{x \rightarrow 3} \frac{9-x^2}{4-\sqrt{x^2+7}} = \lim_{x \rightarrow 3} \frac{9-x^2}{4-\sqrt{x^2+7}} \cdot \frac{4+\sqrt{x^2+7}}{4+\sqrt{x^2+7}} = \lim_{x \rightarrow 3} \frac{9-x^2(4+\sqrt{x^2+7})}{16-(x^2+7)}$

$$= \lim_{x \rightarrow 3} \frac{(9-x^2)(4+\sqrt{x^2+7})}{9-x^2} = \lim_{x \rightarrow 3} (4+\sqrt{x^2+7}) = 4+\sqrt{9+7} = 4+4 = 8$$

(gabungan cara penyelesaian dengan pemfaktoran dan perkalian dengan sekawan)

6.  $\lim_{x \rightarrow 1} \left( \frac{1}{1-x} - \frac{3}{1-x^3} \right) = \dots$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

**Jawab:**

$$\lim_{x \rightarrow 1} \left( \frac{1}{1-x} - \frac{3}{1-x^3} \right) = \lim_{x \rightarrow 1} \left( \frac{(1+x+x^2)}{(1-x)(1+x+x^2)} - \frac{3}{1-x^3} \right) = \lim_{x \rightarrow 1} \left( \frac{(1+x+x^2)-3}{(1-x)(1+x+x^2)} \right)$$

$$= \lim_{x \rightarrow 1} \left( \frac{x^2+x-2}{(1-x)(1+x+x^2)} \right) = \lim_{x \rightarrow 1} \left( \frac{(x+2)(x-1)}{(1-x)(1+x+x^2)} \right)$$

$$= \lim_{x \rightarrow 1} \frac{(x+2)}{(1+x+x^2)} = \frac{1+2}{1+1+1^2} = \frac{3}{3} = 1$$

7.  $\lim_{x \rightarrow 0} \frac{x^2}{1 - \sqrt[3]{1+x^2}} = \dots$

**Jawab:**

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{x^2}{1 - \sqrt[3]{1+x^2}} &= \lim_{x \rightarrow 0} \frac{x^2}{(1 - \sqrt[3]{1+x^2}) \cdot \left(1 + \sqrt[3]{1+x^2} + (\sqrt[3]{1+x^2})^2\right)} = \lim_{x \rightarrow 0} \frac{x^2 \left(1 + \sqrt[3]{1+x^2} + (\sqrt[3]{1+x^2})^2\right)}{1 - (1+x^2)} \\ &= \lim_{x \rightarrow 0} \frac{x^2 \left(1 + \sqrt[3]{1+x^2} + (\sqrt[3]{1+x^2})^2\right)}{-x^2} = \lim_{x \rightarrow 0} - \left(1 + \sqrt[3]{1+x^2} + (\sqrt[3]{1+x^2})^2\right) \\ &= -(1+1+1) = -3 \end{aligned}$$

8. Jika  $\lim_{x \rightarrow n} (x+1) = \lim_{x \rightarrow n} (2x-3)$ , maka tentukan nilai dari  $\lim_{x \rightarrow n} (x^2 - 16)$

**Jawab:**

$$\lim_{x \rightarrow n} (x+1) = \lim_{x \rightarrow n} (2x-3) \Rightarrow n+1 = 2n-3 \Rightarrow n = 4 \text{ maka}$$

$$\lim_{x \rightarrow n} (x^2 - 16) = \lim_{x \rightarrow 4} (x^2 - 16) = 4^2 - 16 = 16 - 16 = 0$$

9. Jika  $\lim_{x \rightarrow 2} \frac{2x^2 + 5x + 2}{x^2 + ax - 10} = \frac{3}{7}$ , maka nilai  $a$  adalah ...

**Jawab:**

$$\lim_{x \rightarrow 2} \frac{2x^2 + 5x + 2}{x^2 + ax - 10}, \text{ karena ketika disubstitusi pembilang bernilai 0, sedangkan nilai}$$

limitnya adalah  $\frac{3}{7}$ , maka penyebut dipastikan bernilai 0. Sehingga diperoleh

$$(-2)^2 - 2a - 10 = 0$$

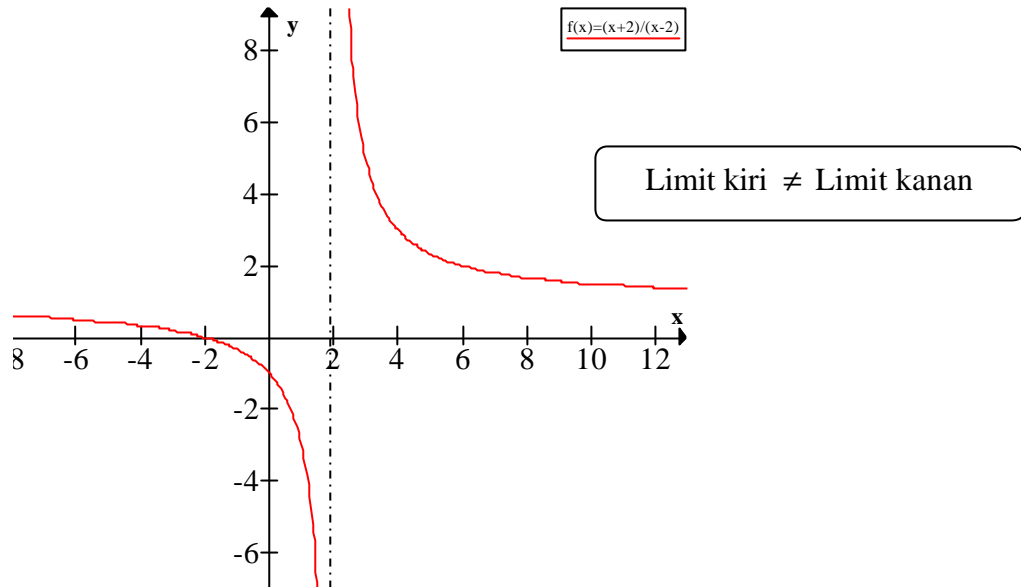
$$\Rightarrow 4 - 10 = 2a$$

$$\Rightarrow 2a = -6$$

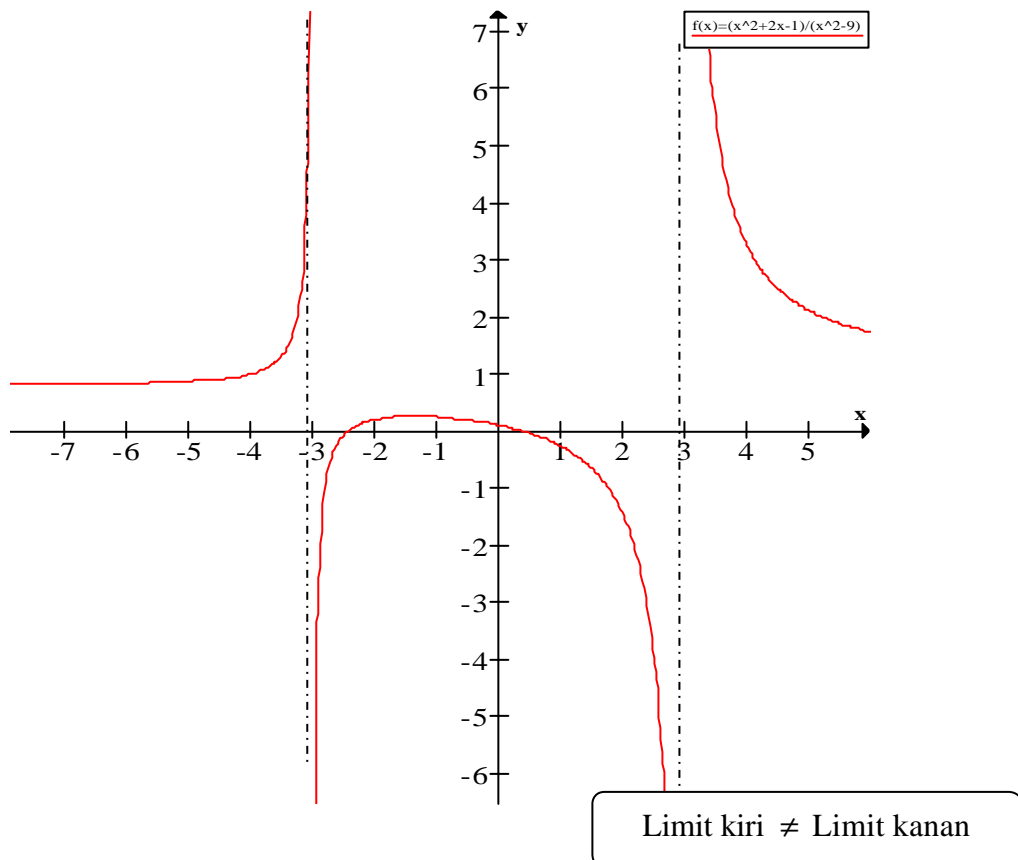
$$\Rightarrow a = -3$$

$$\begin{aligned} \lim_{x \rightarrow 2} \frac{2x^2 + 5x + 2}{x^2 - 3x - 10} &= \lim_{x \rightarrow 2} \frac{(x+2)(2x+1)}{(x+2)(x-5)} = \lim_{x \rightarrow 2} \frac{2x+1}{x-5} \\ &= \frac{2(-2)+1}{-2-5} = \frac{-3}{-7} = \frac{3}{7} \end{aligned}$$

10.  $\lim_{x \rightarrow 2} \frac{x+2}{x-2} = \frac{2+2}{2-2} = \frac{4}{0}$  berarti  $\lim_{x \rightarrow 2} \frac{x+2}{x-2}$  **tidak ada**. Lihat grafiknya berikut ini:



11.  $\lim_{x \rightarrow 3} \frac{x^2 + 2x - 1}{x^2 - 9} = \frac{3^2 + 2 \cdot 3 - 1}{3^2 - 9} = \frac{14}{0}$  berarti  $\lim_{x \rightarrow 3} \frac{x^2 + 2x - 1}{x^2 - 9}$  **tidak ada**. Demikian juga untuk  $\lim_{x \rightarrow -3} \frac{x^2 + 2x - 1}{x^2 - 9}$ , karena  $\lim_{x \rightarrow -3} \frac{x^2 + 2x - 1}{x^2 - 9} = \frac{(-3)^2 + 2(-3) - 1}{(-3)^2 - 9} = \frac{2}{0}$ . Grafiknya adalah:



12. Untuk menentukan nilai  $\lim_{x \rightarrow \infty} f(x)$  adalah dengan SUBSTITUSI,

- Jika  $f(x) = \pm \frac{\infty}{c}$  maka  $\lim_{x \rightarrow \infty} f(x) = \pm \infty$
- Jika  $f(x) = \frac{c}{\infty}$  maka  $\lim_{x \rightarrow \infty} f(x) = 0$
- Jika  $f(x) = \frac{\infty}{\infty}$  (Bentuk Tak Tentu) maka masing2 pembilang dan penyebut dibagi dengan variabel pangkat tertinggi dari penyebut.
- Jika  $f(x) = \infty - \infty$  (Bentuk Tak Tentu) maka masing2 pembilang dan penyebut dikalikan dengan bentuk sekawannya dan dibagi dengan variabel pangkat tertinggi dari penyebut.

**Soal-soal:**

- a.  $\lim_{x \rightarrow \infty} 9 = 9$
- b.  $\lim_{x \rightarrow \infty} 2x + 9 = 2 \cdot \infty + 9 = \infty$
- c.  $\lim_{x \rightarrow \infty} \frac{7x + 9}{8} = \frac{7 \cdot \infty + 9}{8} = \infty$
- d.  $\lim_{x \rightarrow \infty} \frac{6}{x^2 + 1} = \frac{6}{\infty^2 + 1} = \frac{6}{\infty} = 0$

13. Penyelesaian dengan pembagian variabel pangkat tertinggi.

a).  $\lim_{x \rightarrow \infty} \frac{2x}{3x^2 + x - 1} = \frac{\infty}{\infty}$  BTT maka

Variabel Pangkat Tertinggi (VPT) adalah  $x^2$ , maka pembilang dan penyebut dibagi dengan  $x^2$

$$\lim_{x \rightarrow \infty} \frac{2x}{3x^2 + x - 1} = \lim_{x \rightarrow \infty} \frac{\frac{2x}{x^2}}{\frac{3x^2}{x^2} + \frac{x}{x^2} - \frac{1}{x^2}} = \lim_{x \rightarrow \infty} \frac{\frac{2}{x}}{3 + \frac{1}{x} - \frac{1}{x^2}} = \frac{\lim_{x \rightarrow \infty} \frac{2}{x}}{\lim_{x \rightarrow \infty} 3 + \lim_{x \rightarrow \infty} \frac{1}{x} - \lim_{x \rightarrow \infty} \frac{1}{x^2}}$$

$$= \frac{0}{3 + 0 - 0} = \frac{0}{3} = 0$$

Lihat Teorema Limit

b).  $\lim_{x \rightarrow \infty} \frac{2x^2}{3x^2 + x - 1} = \frac{\infty}{\infty}$  BTT, maka



$$\lim_{x \rightarrow \infty} \frac{2x^2}{3x^2 + x - 1} = \lim_{x \rightarrow \infty} \frac{\frac{2x^2}{x^2}}{\frac{3x^2}{x^2} + \frac{x}{x^2} - \frac{1}{x^2}} = \lim_{x \rightarrow \infty} \frac{2}{3 + \frac{1}{x} - \frac{1}{x^2}} = \frac{\lim_{x \rightarrow \infty} 2}{\lim_{x \rightarrow \infty} 3 + \lim_{x \rightarrow \infty} \frac{1}{x} - \lim_{x \rightarrow \infty} \frac{1}{x^2}}$$

$$= \frac{2}{3 + 0 - 0} = \frac{2}{3}$$

c).  $\lim_{x \rightarrow \infty} \frac{2x^3 + 5x}{3x^2 + x - 1} = \frac{\infty}{\infty}$  BTT maka

$$\lim_{x \rightarrow \infty} \frac{2x^3 + 5x}{3x^2 + x - 1} = \lim_{x \rightarrow \infty} \frac{\frac{2x^3}{x^2} + \frac{5x}{x^2}}{\frac{3x^2}{x^2} + \frac{x}{x^2} - \frac{1}{x^2}} = \lim_{x \rightarrow \infty} \frac{2x + \frac{5}{x}}{3 + \frac{1}{x} - \frac{1}{x^2}} = \frac{\infty + 0}{3 + 0 - 0} = \infty$$

14. Penyelesaian dengan perkalian bentuk sekawan kemudian membaginya dengan variabel pangkat tertinggi.

a).  $\lim_{x \rightarrow \infty} (\sqrt{4x^2 - 5x + 1} - \sqrt{4x^2 + 7x - 2}) = \infty - \infty$  BTT, maka

$$\lim_{x \rightarrow \infty} (\sqrt{4x^2 - 5x + 1} - \sqrt{4x^2 + 7x - 2})$$

$$= \lim_{x \rightarrow \infty} (\sqrt{4x^2 - 5x + 1} - \sqrt{4x^2 + 7x - 2}) \cdot \frac{(\sqrt{4x^2 - 5x + 1} + \sqrt{4x^2 + 7x - 2})}{(\sqrt{4x^2 - 5x + 1} + \sqrt{4x^2 + 7x - 2})}$$

$$= \lim_{x \rightarrow \infty} \frac{(4x^2 - 5x + 1) - (4x^2 + 7x - 2)}{\sqrt{4x^2 - 5x + 1} + \sqrt{4x^2 + 7x - 2}}$$

$$= \lim_{x \rightarrow \infty} \frac{-12x + 3}{\sqrt{4x^2 - 5x + 1} + \sqrt{4x^2 + 7x - 2}}$$

$$= \lim_{x \rightarrow \infty} \frac{-12\frac{x}{x} + \frac{3}{x}}{\sqrt{4\frac{x^2}{x^2} - 5\frac{x}{x^2} + \frac{1}{x^2}} + \sqrt{4\frac{x^2}{x^2} + 7\frac{x}{x^2} - \frac{2}{x^2}}}$$

$$= \lim_{x \rightarrow \infty} \frac{-12 + \frac{3}{x}}{\sqrt{4 - \frac{5}{x} + \frac{1}{x^2}} + \sqrt{4 + \frac{7}{x} - \frac{2}{x^2}}}$$

$$= \frac{-12 + 0}{\sqrt{4 - 0 + 0} + \sqrt{4 + 0 - 0}}$$

$$= -\frac{12}{2\sqrt{4}} = -\frac{12}{4} = -3$$

Dikalikan sekawan

Sama nilainya dengan (diambil suku yang memuat pangkat tertinggi dari pembilang dan penyebut):

$$\lim_{x \rightarrow \infty} \frac{-12x}{\sqrt{4x^2} + \sqrt{4x^2}}$$

VPT pembilang adalah  $x$ , dan VPT penyebut  $\sqrt{x^2}$  (setara), maka pembilang dan penyebut dibagi dengan  $x$  (jika dalam akar menjadi  $x^2$ )

Lihat catatan 2

b).  $\lim_{x \rightarrow \infty} (\sqrt{x+6} - \sqrt{x+3}) = \infty - \infty$ , BTT maka:

$$\begin{aligned} \lim_{x \rightarrow \infty} (\sqrt{x+6} - \sqrt{x+3}) &= \lim_{x \rightarrow \infty} (\sqrt{x+6} - \sqrt{x+3}) \frac{(\sqrt{x+6} + \sqrt{x+3})}{(\sqrt{x+6} + \sqrt{x+3})} \\ &= \lim_{x \rightarrow \infty} \frac{3}{(\sqrt{x+6} + \sqrt{x+3})} \\ &= \lim_{x \rightarrow \infty} \frac{3/\sqrt{x}}{(\sqrt{x/x + 6/x} + \sqrt{x/x + 3/x})} \\ &= \lim_{x \rightarrow \infty} \frac{3/\sqrt{x}}{(\sqrt{1 + 6/x} + \sqrt{1 + 3/x})} \\ &= \frac{0}{(\sqrt{1} + \sqrt{1})} \\ &= 0 \end{aligned}$$

#### 15. Beberapa Kesimpulan untuk limit tak hingga:

➤ Jika  $f(x) = \frac{ax^n + bx^{n-1} + \dots}{px^m + qx^{m-1} + \dots}$

maka  $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{ax^n}{px^m} = \begin{cases} 0, & \text{jika } n < m \\ \frac{a}{p}, & \text{jika } n = m \\ \infty, & \text{jika } n > m, \text{ dan } \frac{a}{p} > 0 \\ -\infty, & \text{jika } n > m, \text{ dan } \frac{a}{p} < 0 \end{cases}$

$n$  adalah pangkat tertinggi dari pembilang dan  $m$  adalah pangkat tertinggi dari penyebut.

➤ Jika  $f(x) = \sqrt{ax^2 + bx + c} - \sqrt{px^2 + qx + r}$  maka  $\lim_{x \rightarrow \infty} f(x) = \begin{cases} \infty, & \text{jk } a > p \\ \frac{b-q}{2\sqrt{a}}, & \text{jk } a = p \\ -\infty, & \text{jk } a < p \end{cases}$

➤ Jika  $f(x) = \sqrt{ax+b} - \sqrt{px+q}$  maka  $\lim_{x \rightarrow \infty} f(x) = \begin{cases} \infty, & \text{jk } a > p \\ 0, & \text{jk } a = p \\ -\infty, & \text{jk } a < p \end{cases}$

**Soal-soal:**

a).  $\lim_{x \rightarrow \infty} \frac{5x^3 - x}{x - 3x^3} = \frac{5}{-3}$  (pangkat tertinggi pembilang = pangkat tertinggi penyebut)

b).  $\lim_{x \rightarrow \infty} (\sqrt{9x^2 - 15x + 2} - \sqrt{9x^2 - 7x + 1}) = \frac{-15 - (-7)}{2\sqrt{9}} = \frac{-8}{6} = -\frac{4}{3}$  (nilai  $a = p$ )

c).  $\lim_{x \rightarrow \infty} (\sqrt{2x - 4} - \sqrt{2x + 5}) = 0$  (nilai  $a = p$ )

**16. Teorema Limit**

Untuk  $n \in$  bilangan bulat positif;  $c$  konstanta;  $f$  dan  $g$  fungsi-fungsi dalam  $x$  yang mempunyai limit di  $a$ , maka berlaku:

<p>a. <math>\lim_{x \rightarrow a} c = c</math></p> <p>b. <math>\lim_{x \rightarrow a} x^n = a^n</math></p> <p>c. <math>\lim_{x \rightarrow a} f(x) = f(a)</math></p> <p>d. <math>\lim_{x \rightarrow a} cf(x) = c \lim_{x \rightarrow a} f(x)</math></p> <p>e. <math>\lim_{x \rightarrow a} (f(x) + g(x)) = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)</math></p> <p>f. <math>\lim_{x \rightarrow a} (f(x) - g(x)) = \lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x)</math></p>	<p>g. <math>\lim_{x \rightarrow a} (f(x) \cdot g(x)) = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)</math></p> <p>h. <math>\lim_{x \rightarrow a} \left( \frac{f(x)}{g(x)} \right) = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}</math>; <math>\lim_{x \rightarrow a} g(x) \neq 0</math></p> <p>i. <math>\lim_{x \rightarrow a} (f(x))^n = (\lim_{x \rightarrow a} f(x))^n</math></p> <p>j. <math>\lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow a} f(x)}</math>; <math>\lim_{x \rightarrow a} f(x) \geq 0</math></p>
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**Soal-soal:**

a). a.  $\lim_{x \rightarrow 6} 25 = 25$       b.  $\lim_{x \rightarrow 0} 36 = 36$       c.  $\lim_{x \rightarrow -2} 9 = 9$

b).  $\lim_{x \rightarrow 3} x^4 = 3^4 = 81$

c).  $\lim_{x \rightarrow 2} x^3 - 5x + 7 = 2^3 - 5 \cdot 2 + 7 = 5$

e).  $\lim_{x \rightarrow -2} 5x = 5 \lim_{x \rightarrow -2} x = 5 \cdot (-2) = -10$

f).  $\lim_{x \rightarrow 4} 5x + 3x^2 = \lim_{x \rightarrow 4} 5x + \lim_{x \rightarrow 4} 3x^2 = 5 \cdot 4 + 3 \cdot 4^2 = 20 + 48 = 68$

g).  $\lim_{x \rightarrow 4} 5x - 3x^2 = \lim_{x \rightarrow 4} 5x - \lim_{x \rightarrow 4} 3x^2 = 5 \cdot 4 - 3 \cdot 4^2 = 20 - 48 = -28$

h).  $\lim_{x \rightarrow 1} (5x + 3x^2)(5x - 1) = \lim_{x \rightarrow 1} (5x + 3x^2) \lim_{x \rightarrow 1} (5x - 1) = 8 \cdot 4 = 32$

i).  $\lim_{x \rightarrow 1} \frac{(5x + 3x^2)}{(5x - 1)} = \frac{\lim_{x \rightarrow 1} (5x + 3x^2)}{\lim_{x \rightarrow 1} (5x - 1)} = \frac{8}{4} = 2$

j).  $\lim_{x \rightarrow 1} (5x + 2)^3 = (\lim_{x \rightarrow 1} (5x + 2))^3 = (5 \cdot 1 + 2)^3 = 7^3 = 343$

k).  $\lim_{x \rightarrow 1} \sqrt[3]{5x + 2} = \sqrt[3]{\lim_{x \rightarrow 1} (5x + 2)} = \sqrt[3]{(5 \cdot 1 + 2)} = \sqrt[3]{7}$

$$1). \lim_{x \rightarrow -5} \frac{5x - 3x^2}{2x + 7} = \frac{\lim_{x \rightarrow -5} (5x - 3x^2)}{\lim_{x \rightarrow -5} (2x + 7)} = \frac{\lim_{x \rightarrow -5} 5x - \lim_{x \rightarrow -5} 3x^2}{\lim_{x \rightarrow -5} 2x + \lim_{x \rightarrow -5} 7} = \frac{5 \cdot (-5) - 3 \cdot (-5)^2}{2 \cdot (-5) + 7} = \frac{-25 - 75}{-10 + 7} = \frac{-100}{-3} = \frac{100}{3}$$

## 17. Limit Fungsi Trigonometri

Cara menentukan nilai limit fungsi trigonometri sama dengan limit fungsi aljabar. Beberapa persamaan khusus:

<p>a. <math>\lim_{x \rightarrow 0} \frac{\sin x}{x} = \lim_{x \rightarrow 0} \frac{x}{\sin x} = 1</math></p> <p>b. <math>\lim_{x \rightarrow 0} \frac{\tan x}{x} = \lim_{x \rightarrow 0} \frac{x}{\tan x} = 1</math></p> <p>c. <math>\lim_{x \rightarrow 0} \frac{\sin ax}{bx} = \lim_{x \rightarrow 0} \frac{ax}{\sin bx} = \frac{a}{b}</math></p>	<p>d. <math>\lim_{x \rightarrow 0} \frac{\tan ax}{bx} = \lim_{x \rightarrow 0} \frac{ax}{\tan bx} = \frac{a}{b}</math></p> <p>e. <math>\lim_{x \rightarrow 0} \frac{\tan ax}{\sin bx} = \lim_{x \rightarrow 0} \frac{\sin ax}{\tan bx} = \frac{a}{b}</math></p>
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### Soal-soal:

a).  $\lim_{x \rightarrow 0} \frac{x}{\cos x} = \frac{0}{\cos 0} = \frac{0}{1} = 0$

b).  $\lim_{x \rightarrow \frac{1}{2}p} \sin x + \cos x = \sin \frac{1}{2}p + \cos \frac{1}{2}p = 1 + 0 = 1$

c).  $\lim_{x \rightarrow 0} \frac{\sin 2x}{x} = \lim_{x \rightarrow 0} \frac{\sin 2x}{x} \cdot \frac{2}{2} = 2 \cdot \lim_{2x \rightarrow 0} \frac{\sin 2x}{2x} = 2 \cdot 1 = 2$  (jika  $x \rightarrow 0$  maka  $2x \rightarrow 0$ )

d).  $\lim_{x \rightarrow 0} \frac{3x + \sin 4x}{5x - \tan 2x} = \frac{0}{0}$  BTT, maka

(khusus soal model ini, pembilang dan penyebut dibagi dengan  $x$ )

$$\lim_{x \rightarrow 0} \frac{3x + \sin 4x}{5x - \tan 2x} = \lim_{x \rightarrow 0} \frac{\frac{3x}{x} + \frac{\sin 4x}{x}}{\frac{5x}{x} - \frac{\tan 2x}{x}} = \lim_{x \rightarrow 0} \frac{3 + \frac{\sin 4x}{x}}{5 - \frac{\tan 2x}{x}} = \frac{\lim_{x \rightarrow 0} 3 + \lim_{x \rightarrow 0} \frac{\sin 4x}{x}}{\lim_{x \rightarrow 0} 5 - \lim_{x \rightarrow 0} \frac{\tan 2x}{x}} = \frac{3 + 4}{5 - 2} = \frac{7}{3}$$

e).  $\lim_{x \rightarrow 0} \frac{1 - \cos 4x}{x \sin x} = \frac{0}{0}$  BTT, maka

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{1 - \cos 4x}{x \sin x} &= \lim_{x \rightarrow 0} \frac{1 - \cos 4x}{x \sin x} \cdot \frac{1 + \cos 4x}{1 + \cos 4x} = \lim_{x \rightarrow 0} \frac{1 - \cos^2 4x}{(x \sin x)(1 + \cos 4x)} = \lim_{x \rightarrow 0} \frac{\sin^2 4x}{(x \sin x)(1 + \cos 4x)} \\ &= \lim_{x \rightarrow 0} \frac{\sin 4x \cdot \sin 4x}{x \sin x} \cdot \frac{1}{1 + \cos 4x} \cdot \frac{4x \cdot 4x}{4x \cdot 4x} = \lim_{x \rightarrow 0} \frac{\sin 4x}{4x} \cdot \frac{\sin 4x}{4x} \cdot \frac{4x}{\sin x} \cdot \frac{1}{(1 + \cos 4x)} \cdot \frac{4x}{x} \\ &= 1 \cdot 1 \cdot 4 \cdot \frac{1}{2} \cdot 4 = 8 \end{aligned}$$

f).  $\lim_{x \rightarrow \frac{p}{2}} \left( \frac{\cos x}{x - \frac{p}{2}} \right) = \frac{0}{0}$  BTT, maka

Diketahui rumus trigonometri:  $\cos x = \sin \left( \frac{p}{2} - x \right)$

$$\lim_{x \rightarrow \frac{p}{2}} \left( \frac{\cos x}{x - \frac{p}{2}} \right) = \lim_{x \rightarrow \frac{p}{2}} \frac{\sin \left( \frac{p}{2} - x \right)}{x - \frac{p}{2}} = \lim_{x \rightarrow \frac{p}{2}} \frac{\sin \left( x - \frac{p}{2} \right)}{x - \frac{p}{2}} = \lim_{x \rightarrow \frac{p}{2}} \frac{-\sin \left( x - \frac{p}{2} \right)}{x - \frac{p}{2}} = -\lim_{x \rightarrow \frac{p}{2}} \frac{\sin \left( x - \frac{p}{2} \right)}{x - \frac{p}{2}} = -1$$

g).  $\lim_{x \rightarrow a} \frac{\cos x - \cos a}{x - a} = \frac{0}{0}$ , BTT maka

$$\begin{aligned} \lim_{x \rightarrow a} \frac{\cos x - \cos a}{x - a} &= \lim_{x \rightarrow a} \frac{-2 \sin \frac{1}{2}(x+a) \sin \frac{1}{2}(x-a)}{x - a} = -2 \lim_{x \rightarrow a} \sin \frac{1}{2}(x+a) \lim_{x \rightarrow a} \frac{\sin \frac{1}{2}(x-a)}{x - a} \\ &= -2 \sin a \cdot \frac{1}{2} = -\sin a \end{aligned}$$

h).  $\lim_{x \rightarrow 1} \frac{x^3 - (a+1)x^2 + ax}{(x^2 - 1) + \tan(x-1)} = \frac{0}{0}$ , BTT maka

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{x^3 - (a+1)x^2 + ax}{(x^2 - 1) + \tan(x-1)} &= \lim_{x \rightarrow 1} \frac{x(x^2 - (a+1)x + a)}{(x-1)(x+1) + \tan(x-1)} = \lim_{x \rightarrow 1} \frac{x(x-1)(x-a)}{(x-1)(x+1) + \tan(x-1)} \\ &= \lim_{x \rightarrow 1} \frac{x(x-a)}{(x+1) + \frac{\tan(x-1)}{(x-1)}} = \frac{\lim_{x \rightarrow 1} x(x-a)}{\lim_{x \rightarrow 1} (x+1) + \lim_{(x-1) \rightarrow 0} \frac{\tan(x-1)}{(x-1)}} = \frac{1-a}{2+1} = \frac{1}{3}(1-a) \end{aligned}$$

i).  $\lim_{x \rightarrow y} \left[ \frac{\tan x - \tan y}{1 - \frac{x}{y} + \left(1 - \frac{x}{y}\right) \tan x \tan y} \right] = \frac{0}{0}$  BTT maka

$$\begin{aligned} \lim_{x \rightarrow y} \left[ \frac{\tan x - \tan y}{1 - \frac{x}{y} + \left(1 - \frac{x}{y}\right) \tan x \tan y} \right] &= \lim_{x \rightarrow y} \frac{1}{\left(1 - \frac{x}{y}\right)} \left[ \frac{\tan x - \tan y}{1 + \tan x \tan y} \right] = \lim_{x \rightarrow y} \frac{1}{\left(\frac{y-x}{y}\right)} \tan(x-y) \\ &= \lim_{x \rightarrow y} \frac{y}{(y-x)} \tan(x-y) = \lim_{x \rightarrow y} \frac{-y}{(x-y)} \tan(x-y) \\ &= -y \left[ \lim_{(x-y) \rightarrow 0} \frac{\tan(x-y)}{(x-y)} \right] \\ &= -y \end{aligned}$$

18. Apakah fungsi  $f(x) = 2x + 1$ , kontinu di  $x = 1$  ?

**Jawab:**

**Kekontinuan Suatu Fungsi**

Suatu fungsi  $f$  dikatakan kontinu pada  $x = a$  jika:

- a.  $f(a)$  ada
- b.  $\lim_{x \rightarrow a} f(x)$  ada
- c.  $\lim_{x \rightarrow a} f(x) = f(a)$

Ciri:  
Grafiknya merupakan lengkungan (kurva)  
yang tidak terputus.

Fungsi  $f(x) = 2x + 1$ , kontinu di  $x = 1$  karena  $\lim_{x \rightarrow 1} (2x + 1) = 3 = f(1)$

19. Apakah fungsi  $f(x) = \begin{cases} x^2 - 9 & ; x \neq 3 \\ 3 & ; x = 3 \end{cases}$ , kontinu di  $x = 3$  ?

**Jawab:**

Fungsi  $f(x) = \begin{cases} x^2 - 9 & ; x \neq 3 \\ 3 & ; x = 3 \end{cases}$  maka  $f(x)$  tidak kontinu di  $x = 3$ , karena

- a.  $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} = \lim_{x \rightarrow 3} \frac{(x - 3)(x + 3)}{(x - 3)} = \lim_{x \rightarrow 3} (x + 3) = 3 + 3 = 6$
- b.  $f(3) = 3$

maka  $\lim_{x \rightarrow 3} f(x) \neq f(3)$

20. Tentukan nilai  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$  untuk fungsi  $f(x) = 2x^3$

**Jawab:**

$$f(x) = 2x^3 \Rightarrow f(x+h) = 2(x+h)^3 = 2(x^3 + 3x^2h + 3xh^2 + h^3) = 2x^3 + 6x^2h + 6xh^2 + 2h^3$$

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h \rightarrow 0} \frac{(2x^3 + 6x^2h + 6xh^2 + 2h^3) - 2x^3}{h} = \lim_{h \rightarrow 0} \frac{6x^2h + 6xh^2 + 2h^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(6x^2 + 6xh + 2h^2)}{h} = \lim_{h \rightarrow 0} (6x^2 + 6xh + 2h^2) = 6x^2 + 0 + 0 = 6x^2 \end{aligned}$$

21. Tentukan nilai  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$  untuk fungsi  $f(x) = x^2 + 3x$

**Jawab:**

$$f(x) = x^2 + 3x$$

$$\begin{aligned}\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h \rightarrow 0} \frac{[(x+h)^2 + 3(x+h)] - [x^2 + 3x]}{h} = \lim_{h \rightarrow 0} \frac{[x^2 + 2xh + h^2 + 3x + 3h] - [x^2 + 3x]}{h} \\ &= \lim_{h \rightarrow 0} \frac{2xh + h^2 + 3h}{h} = \lim_{h \rightarrow 0} \frac{h(2x + h + 3)}{h} = \lim_{h \rightarrow 0} (2x + h + 3) = 2x + 0 + 3 = 2x + 3\end{aligned}$$

## 22. Limit Barisan Bilangan

$$\begin{array}{ll} 1. \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e & 3. \lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^x = e^{-1} \\ 2. \lim_{x \rightarrow \infty} (1+x)^{\frac{1}{x}} = e & 4. \lim_{x \rightarrow \infty} (1-x)^{\frac{1}{x}} = e^{-1} \end{array}$$

Ket:  $e = 2,7182818... = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \dots$  (bilangan Euler)

### Soal-soal:

$$a. \lim_{x \rightarrow \infty} \left(\frac{x}{x+1}\right)^{x+1} = \lim_{x \rightarrow \infty} \left(\frac{x+1-1}{x+1}\right)^{x+1} = \lim_{x \rightarrow \infty} \left(\frac{x+1}{x+1} - \frac{1}{x+1}\right)^{x+1} = \lim_{x \rightarrow \infty} \left(1 - \frac{1}{x+1}\right)^{x+1} = e^{-1}$$

Atau

$$\begin{aligned}\lim_{x \rightarrow \infty} \left(\frac{x}{x+1}\right)^{x+1} &= \lim_{x \rightarrow \infty} \left(\frac{x+1-1}{x+1}\right)^{x+1} = \lim_{x \rightarrow \infty} \left(\frac{x+1}{x+1} - \frac{1}{x+1}\right)^{x+1} = \lim_{x \rightarrow \infty} \left(1 - \frac{1}{x+1}\right)^{x+1} \\ &= \lim_{x \rightarrow \infty} \left\{ \left(1 - \frac{1}{x+1}\right)^{-(x+1)} \right\}^{-1} = \lim_{x \rightarrow \infty} \left\{ \left(1 + \frac{1}{-(x+1)}\right)^{-(x+1)} \right\}^{-1} = e^{-1}\end{aligned}$$

$$b. \lim_{x \rightarrow \infty} (1-3x)^{\frac{1}{x}} = \lim_{x \rightarrow \infty} (1-3x)^{\frac{1}{3x} \cdot 3} = \lim_{x \rightarrow \infty} \left[(1-3x)^{\frac{1}{3x}}\right]^{-3} = \left[\lim_{x \rightarrow \infty} (1+(-3x))^{\frac{1}{-3x}}\right]^{-3} = e^{-3}$$

$$\begin{aligned}c. \lim_{x \rightarrow \infty} \left(1 + \frac{2}{3+x}\right)^{-2x} &= \lim_{x \rightarrow \infty} \left(1 + \frac{2}{3+x}\right)^{\left(\frac{3+x}{2}\right)^{-4} \cdot 6} = \lim_{x \rightarrow \infty} \left[\left(1 + \frac{2}{3+x}\right)^{\left(\frac{3+x}{2}\right)^{-4}}\right]^{-4} \left[\left(1 + \frac{2}{3+x}\right)^6\right] \\ &= \left[\lim_{x \rightarrow \infty} \left(1 + \frac{2}{3+x}\right)^{\left(\frac{3+x}{2}\right)^{-4}}\right]^{-4} \cdot \lim_{x \rightarrow \infty} \left(1 + \frac{2}{3+x}\right)^6 = e^{-4} \cdot (1+0)^6 = e^{-4}\end{aligned}$$

**Catatan:**

- a.  $a^2 - b^2 = (a - b)(a + b)$   
 b.  $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$   
 c.  $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$   
 d.  $(a + b)^2 = a^2 + 2ab + b^2$   
 e.  $(a - b)^2 = a^2 - 2ab + b^2$   
 f.  $(\sqrt{a})^2 = \sqrt{a} \cdot \sqrt{a} = a$   
 g.  $(\sqrt{a + b})^2 = \sqrt{a + b} \cdot \sqrt{a + b} = a + b$

**Bentuk Sekawan:**

- a.  $\sqrt{a} - \sqrt{b}$  sekawannya  $\sqrt{a} + \sqrt{b}$   
 b.  $a + \sqrt{b - c}$  sekawannya  $a - \sqrt{b - c}$   
 c.  $a\sqrt{b} - c$  sekawannya  $a\sqrt{b} + c$   
 d.  $\sqrt{a + b} + \sqrt{c - d}$  sekawannya  $\sqrt{a + b} - \sqrt{c - d}$   
 e.  $\sqrt{a + b} - c$  sekawannya  $\sqrt{a + b} + c$

dan lain sebagainya..

**Catatan 2:**

- a.  $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$       b.  $\frac{\sqrt{a}}{x} = \frac{\sqrt{a}}{\sqrt{x^2}} = \sqrt{\frac{a}{x^2}}$       c.  $\frac{\sqrt{ax + b}}{x^2} = \frac{\sqrt{ax + b}}{\sqrt{x^4}} = \sqrt{\frac{ax + b}{x^4}} = \sqrt{\frac{ax}{x^4} + \frac{b}{x^4}}$

dan lain-lain.

**Keterangan:**

Sebagian materi adalah materi pengayaan, tidak semuanya dipelajari di kelas.



# Soal-Soal Latihan

Kerjakan soal-soal berikut, bila perlu gambarlah grafiknya.

1. Jika  $f(x) = \begin{cases} 2; & jk \ x \leq 0 \\ x^2; & jk \ x > 0 \end{cases}$ , tentukan: a.  $\lim_{x \rightarrow 0^-} f(x)$ , b.  $\lim_{x \rightarrow 0^+} f(x)$ , c.  $\lim_{x \rightarrow 0} f(x)$  jk ada.

2. Jika  $f(x) = \begin{cases} 3x+2; & jk \ x < 1 \\ x+4; & jk \ x \geq 1 \end{cases}$ , tentukan: a.  $\lim_{x \rightarrow 1^-} f(x)$ , b.  $\lim_{x \rightarrow 1^+} f(x)$ , c.  $\lim_{x \rightarrow 1} f(x)$ .

3. Jika  $f(x) = \begin{cases} 4x+1; & jk \ x \leq 1 \\ 2x^2+3; & jk \ x > 1 \end{cases}$ , tentukan: a.  $\lim_{x \rightarrow 1^-} f(x)$ , b.  $\lim_{x \rightarrow 1^+} f(x)$ , c.  $\lim_{x \rightarrow 1} f(x)$ .

4. Jika  $f(x) = \begin{cases} -1; & jk \ x < -1 \\ 0; & jk \ x = -1 \\ 1; & jk \ x > -1 \end{cases}$ , tentukan: a.  $\lim_{x \rightarrow -1^-} f(x)$ , b.  $\lim_{x \rightarrow -1^+} f(x)$ , c.  $\lim_{x \rightarrow -1} f(x)$ .

5. Ditetapkan  $f(x) = \begin{cases} 2; & jk \ x < -1 \\ 1-x; & jk \ -1 \leq x < 1 \\ 0; & jk \ x \geq 1 \end{cases}$

Selidiki apakah ada nilai limit fungsi berikut: a.  $\lim_{x \rightarrow -1} f(x)$       b.  $\lim_{x \rightarrow 1} f(x)$

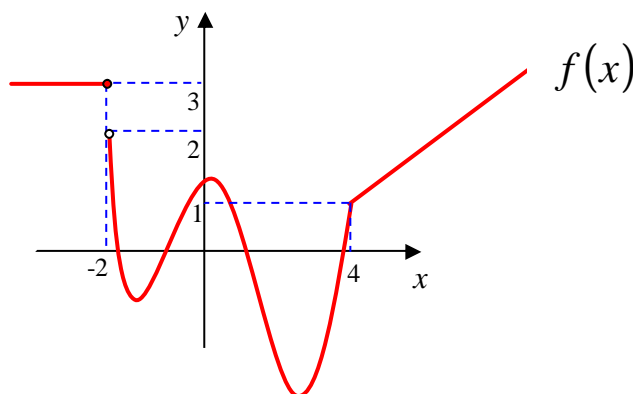
6. Tentukan nilai dari: a.  $\lim_{x \rightarrow 1^+} \sqrt{x-1}$       b.  $\lim_{x \rightarrow -1^+} x^2$       c.  $\lim_{x \rightarrow 0^+} \frac{1}{x^2}$

7. Tentukan nilai dari: a.  $\lim_{x \rightarrow 4^-} \sqrt{4x}$       b.  $\lim_{x \rightarrow -2^-} x$       c.  $\lim_{x \rightarrow 0^-} \frac{3}{2x}$

8. Diketahui fungsi  $f(x) = \sqrt{x}$ . Tentukan nilai berikut jika ada! (cari limit kiri dan limit kanan). a.  $\lim_{x \rightarrow 1} f(x)$       b.  $\lim_{x \rightarrow 3} f(x)$       c.  $\lim_{x \rightarrow 16} f(x)$       d.  $\lim_{x \rightarrow 0} f(x)$

9. Selidikilah, apakah  $\lim_{x \rightarrow 0} \frac{1}{x}$  ada? (cari limit kiri dan limit kanan).

10. Tentukan  $\lim_{x \rightarrow -2} f(x)$  dan  $\lim_{x \rightarrow 4} f(x)$  dari gambar berikut:



**Carilah Nilai Limit Berikut:**

11.  $\lim_{x \rightarrow 5} 1000$

12.  $\lim_{x \rightarrow 1} 12345$

13.  $\lim_{x \rightarrow -2} 2x + 5$

14.  $\lim_{x \rightarrow 0} 3x^2 + 5x - 10$

15.  $\lim_{x \rightarrow -3} (x-4)(x+1)$

16.  $\lim_{x \rightarrow -3} [(4x-7)\sqrt[3]{(3-x)}]$

17.  $\lim_{x \rightarrow 4} \frac{x}{x+2}$

18.  $\lim_{x \rightarrow 4} \left[ \left( \frac{3x-1}{x+2} \right) \sqrt{\frac{x}{x-3}} \right]$

19.  $\lim_{x \rightarrow 0} \frac{3x^2 - 5x + 10}{x^3 + 6x - 45}$

20.  $\lim_{x \rightarrow 2} \frac{6x+9}{7x-10}$

21.  $\lim_{x \rightarrow 9} \sqrt{4x-11}$

22.  $\lim_{x \rightarrow 4} \sqrt{x^2 - 7}$

23.  $\lim_{x \rightarrow 1} \sqrt{\frac{x^2 - 6}{-x^3}}$

24.  $\lim_{x \rightarrow 2} \sqrt{\frac{x^2 + 3x + 6}{x^3 + 1}}$

25.  $\lim_{x \rightarrow 2} \frac{1}{x-2}$

26.  $\lim_{x \rightarrow 4} \frac{x+4}{x^2 - 2x - 24}$

40. Jika  $\lim_{x \rightarrow n} (x+1) = \lim_{x \rightarrow n} (2x-3)$ , maka tentukan nilai dari:  $\lim_{x \rightarrow n} (x^2 - 16)$

41. Jika  $\lim_{x \rightarrow 7} \frac{x^2 - 6x - 7}{x^2 - 10x + 21} = a$ , berapakah nilai dari  $\lim_{x \rightarrow a} \frac{4x^2 - 7x - 2}{3 - \sqrt{4x+1}}$ ?

27.  $\lim_{x \rightarrow -1} \frac{\sqrt{x+5}}{x^2 - 2x - 24}$

28.  $\lim_{x \rightarrow -3} \frac{\sqrt{6-x}}{\sqrt{x+6}}$

29.  $\lim_{x \rightarrow 3} \frac{x-3}{x}$

30.  $\lim_{x \rightarrow 2} \left( \frac{2x-3}{x} + \frac{2x}{6-7x} \right)$

31.  $\lim_{x \rightarrow 2} \left( \frac{9x}{8+5x} + \sqrt{5x+14} \right)$

32.  $\lim_{x \rightarrow 5} \frac{(x-3)(x-5)}{2x-1}$

33.  $\lim_{x \rightarrow 7} \frac{(x-3)(x-5)}{\sqrt{2x+2} + 5x}$

34.  $\lim_{x \rightarrow 1} \frac{\sqrt{x+3} + \sqrt{5x+4}}{\sqrt{15-6x} - \sqrt{2x-1}}$

35.  $\lim_{x \rightarrow 4} (\sqrt{8-2x} + \sqrt{-5x+5})$

36.  $\lim_{x \rightarrow 3} (\sqrt{2x^2 + 3x - 2} - \sqrt{2x^2 - 4x + 3})$

37.  $\lim_{x \rightarrow a} \frac{x+9}{2x-1}$

38.  $\lim_{x \rightarrow m} \frac{7x}{m}$

39.  $\lim_{x \rightarrow n} \frac{x^2 + x}{n}$

42. Jika  $\lim_{x \rightarrow 2} \frac{2x^2 + 5x + 2}{x^2 + ax - 10} = \frac{3}{7}$ , maka  $a = \dots$

43. Jika  $\lim_{x \rightarrow 3} \frac{3x^2 + ax - 1}{x^2 - ax - 30} = \frac{11}{13}$ , maka  $a = \dots$

44.  $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1}$

49.  $\lim_{x \rightarrow -3} \frac{2x+6}{x^2+x-6}$

45.  $\lim_{x \rightarrow 1} \frac{x-1}{1-\sqrt{x}}$

50.  $\lim_{x \rightarrow 0} \frac{3x^2 - 5x}{x}$

46.  $\lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{x-1}$

51.  $\lim_{x \rightarrow 0} \frac{\sqrt{x}}{x+\sqrt{x}}$

47.  $\lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{1-x}$

52.  $\lim_{x \rightarrow 4} \frac{x-4}{\sqrt{x}-2}$

48.  $\lim_{x \rightarrow 1} \frac{x^2 + 5x - 6}{x - 1}$

53. Dengan menyederhanakan lebih dahulu (menyamakan penyebut), hitunglah:

a.  $\lim_{x \rightarrow 0} \left( \frac{1}{x^2 - x} + \frac{1}{x} \right)$     b.  $\lim_{x \rightarrow 0} \left( \frac{2}{x^2 - 1} - \frac{1}{x - 1} \right)$     c.  $\lim_{x \rightarrow 1} \left( \frac{1}{1 - x} - \frac{3}{1 - x^3} \right)$

d.  $\lim_{x \rightarrow 2} \left( \frac{2}{x^2 - 4} - \frac{3}{x^2 + 2x - 8} \right)$

54.  $\lim_{x \rightarrow -1} \frac{2x + 2}{x^2 - 3x - 4}$

61.  $\lim_{x \rightarrow 0} \frac{x^{n+3} + 6x^{n+1} - x^n}{x^{n+4} + 2x^n}$

55.  $\lim_{x \rightarrow 2} \frac{3x^2 - 6x}{x - 2}$

62.  $\lim_{x \rightarrow 1} \frac{2x^3 + 3x^2 - 2x - 3}{x^2 - 1}$

56.  $\lim_{x \rightarrow 3} \frac{(x-2)^2 - 1}{x-3}$  (Ebtanas IPS 99)

63.  $\lim_{x \rightarrow 2} \frac{x^3 + x^2 - 8x + 4}{x^3 - 2x^2 - x + 2}$

57.  $\lim_{x \rightarrow \frac{1}{2}} \frac{2x-1}{2x^2+3x-2}$

64.  $\lim_{x \rightarrow 2} \frac{x^3 + x^2 - 6x}{x^3 - 2x^2 + 6x - 12}$

58.  $\lim_{x \rightarrow 1} \frac{x^2 + 3x - 4}{x^2 - 2x + 1}$

65.  $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2}$

59.  $\lim_{x \rightarrow 0} \frac{x^2 + 2x}{x^3 + x^2 + 3x}$

66.  $\lim_{x \rightarrow 1} \frac{x^3 - 1}{1 - x}$

60.  $\lim_{x \rightarrow 0} \frac{x^4 - 6x^2}{x^3 + 2x^2}$

67.  $\lim_{x \rightarrow 3} \frac{x-3}{x^3 - 27}$

68.  $\lim_{x \rightarrow 4} \frac{4-x}{x^3 - 64}$

$$69. \lim_{x \rightarrow 1} \frac{x-1}{\sqrt[3]{x}-1}$$

$$70. \lim_{x \rightarrow \frac{3}{2}} \sqrt[3]{\frac{8x^3-27}{4x^2-9}}$$

\*\*

$$71. \lim_{x \rightarrow 4} \frac{x^2-2x-8}{\sqrt{x}-2}$$

$$72. \lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{x^4-x}$$

73. Diketahui  $g(x) = \sqrt{1+2x}$ , maka nilai

$$\lim_{x \rightarrow 0} \frac{g(1+x) - g(1-x)}{x} = \dots$$

$$74. \lim_{x \rightarrow 1} \frac{x-1}{2-\sqrt{3x+1}}$$

$$75. \lim_{x \rightarrow 2} \frac{x^2-3x+2}{\sqrt{2x+5}-\sqrt{x+7}}$$

$$76. \lim_{x \rightarrow 6} \frac{\sqrt{x-2}-\sqrt{10-x}}{\sqrt{6x}-\sqrt{5x+6}}$$

$$77. \lim_{x \rightarrow 3} \frac{\sqrt{x+2}-\sqrt{2x-1}}{\sqrt{2x-3}-\sqrt{x}}$$

$$78. \lim_{x \rightarrow 1} \frac{\sqrt{3-x}-\sqrt{3x-1}}{\sqrt{5x-1}-\sqrt{x+3}}$$

$$79. \lim_{x \rightarrow 0} \frac{\sqrt{x^2+2x+3}-\sqrt{x^2-2x+3}}{\sqrt{x+3}-\sqrt{3-x}}$$

$$80. \lim_{x \rightarrow 3} \frac{\sqrt{5x+1}-4}{x^2-9}$$

$$81. \lim_{x \rightarrow 10} \frac{\sqrt{x-1}-3}{x-10}$$

$$82. \lim_{x \rightarrow 3} \frac{x-\sqrt{2x+3}}{x^2-9}$$

$$83. \lim_{x \rightarrow 1} \frac{\sqrt{x^2+3}-x-1}{1-x^2}$$

$$84. \lim_{x \rightarrow 0} \left( \frac{\sqrt{x^2+x}}{x\sqrt{x}} - \frac{1}{x} \right)$$

$$85. \lim_{x \rightarrow -1} \left( \frac{x^2}{\sqrt{x+1}} - \frac{1}{\sqrt{x+1}} \right)$$

$$86. \lim_{x \rightarrow -3} \frac{9-x^2}{4-\sqrt{x^2+7}}$$

$$87. \lim_{x \rightarrow 0} \frac{2x^2-5x}{3-\sqrt{9+x}}$$

$$88. \lim_{x \rightarrow 5} \frac{4-\sqrt{x^2-9}}{5-x}$$

$$89. \lim_{x \rightarrow 3} \frac{\sqrt{x+4}-\sqrt{2x+1}}{x-3}$$

$$90. \lim_{x \rightarrow 5} \frac{\sqrt{x+4}-\sqrt{x-4}}{\sqrt{x}-\sqrt{5}}$$

$$91. \lim_{x \rightarrow 2} \frac{\sqrt{x}-2}{\sqrt{2x+1}+\sqrt{2-x}}$$

$$92. \lim_{x \rightarrow 1} \frac{\sqrt{3+x}+\sqrt{5x-1}}{\sqrt{3+x}-\sqrt{5x-1}}$$

$$93. \lim_{x \rightarrow 2} \frac{2x-\sqrt{x+3}}{\sqrt{3x+6}-x}$$

$$94. \lim_{x \rightarrow 3} \frac{x^2-5x+6}{\sqrt{3-x}-\sqrt{x-3}}$$

$$95. \lim_{x \rightarrow 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x}$$

$$96. \lim_{x \rightarrow 0} \frac{4x}{\sqrt{1+2x}-\sqrt{1-2x}}$$

$$97. \lim_{x \rightarrow 1} \frac{1-x}{\sqrt{1-x}-\sqrt{x-1}}$$

$$98. \lim_{x \rightarrow 0} \frac{x^2}{1-\sqrt[3]{1+x^2}}$$

$$99. \lim_{x \rightarrow 1} \frac{x-3\sqrt{x}+2}{2x-8\sqrt{x}+6}$$

$$100. \lim_{x \rightarrow p} \frac{x\sqrt{x} - p\sqrt{p}}{\sqrt{x} - \sqrt{p}}$$

$$102. \lim_{x \rightarrow 1} \frac{x^n - 1}{x - 1} \quad **$$

$$101. \lim_{x \rightarrow 1} \frac{\sqrt[3]{x^2} - 2\sqrt[3]{x} + 1}{(x-1)^2} \quad **$$

$$103. \text{ Diketahui } f(x) = 3x^2 - 2x, \text{ tentukan } \lim_{x \rightarrow 2} \frac{\left(f(x) - \frac{1}{4} \cdot f(2)(x+2)\right)}{x-2}$$

$$104. \text{ Diketahui } f(x) = \frac{3}{x^2}, \text{ tentukan } \lim_{x \rightarrow 2} \frac{(f(x) - f(2))}{x-2}$$

**Hitunglah nilai dari limit fungsi berikut:**

$$105. \lim_{x \rightarrow \infty} \frac{2}{x}$$

$$117. \lim_{x \rightarrow \infty} \frac{6-8x}{x+5}$$

$$106. \lim_{x \rightarrow \infty} \frac{6}{5x^{10}}$$

$$118. \lim_{x \rightarrow \infty} \frac{10+3x}{9x-5}$$

$$107. \lim_{x \rightarrow \infty} \frac{-9}{2x^{25}}$$

$$119. \lim_{x \rightarrow \infty} \frac{10+3x}{3-9x}$$

$$108. \lim_{x \rightarrow \infty} \frac{7}{2x^3 + 5x}$$

$$120. \lim_{x \rightarrow \infty} \left(\frac{3x-2}{5+x}\right)^3$$

$$109. \lim_{x \rightarrow \infty} \frac{-3}{x^3 - 20}$$

$$121. \lim_{x \rightarrow \infty} \frac{7-5x^2}{3x+12x^2}$$

$$110. \lim_{x \rightarrow \infty} 4x + 99$$

$$122. \lim_{x \rightarrow \infty} \frac{5x^3 - 11x^2}{3x + 12x^3}$$

$$111. \lim_{x \rightarrow \infty} x^2 + 9x - 15$$

$$123. \lim_{x \rightarrow \infty} \frac{(5x-1)(2x+3)}{(3+12x)(x-1)}$$

$$112. \lim_{x \rightarrow \infty} \frac{3x}{100}$$

$$124. \lim_{x \rightarrow \infty} \frac{x^2 + 5x - 3}{(3-x)(x-1)}$$

$$113. \lim_{x \rightarrow \infty} \frac{7x+4}{55}$$

$$125. \lim_{x \rightarrow \infty} \frac{(x-1)(x-3)}{2x^2 + 3x - 15}$$

$$114. \lim_{x \rightarrow \infty} \frac{x^2 - 25}{12}$$

$$126. \lim_{x \rightarrow \infty} \frac{(4x-1)^3}{2x^3 - 1}$$

$$115. \lim_{x \rightarrow \infty} \frac{x+5}{2x-1}$$

$$127. \lim_{x \rightarrow \infty} \frac{4(2x+3)^3}{3x^3 + 5x}$$

$$116. \lim_{x \rightarrow \infty} \frac{4x-3}{2x+5}$$

$$128. \lim_{x \rightarrow \infty} \frac{\sqrt{4x^4 + 8x}}{2x^2}$$

$$129. \lim_{x \rightarrow \infty} \frac{4x^2 + 3x - 1}{3x^2 + 5x - 2}$$

$$130. \lim_{x \rightarrow \infty} \frac{x + 3x^3}{3x^3 - 2}$$

$$131. \lim_{x \rightarrow \infty} \frac{(2x - 5)^4}{(3x^2 + 2)^2}$$

$$132. \lim_{x \rightarrow \infty} \frac{6x + x^3 - 5x^4}{x^3 - 2x^4}$$

$$133. \lim_{x \rightarrow \infty} \frac{x(2x+1)^2}{5x - 4x^3}$$

$$134. \lim_{x \rightarrow \infty} \frac{2(x-1)^3}{x^3 + 1}$$

$$135. \lim_{x \rightarrow \infty} \frac{6x + 2x^3}{(x-3)(x+1)}$$

$$136. \lim_{x \rightarrow \infty} \frac{(x^2 - 2)(x^2 + 2)}{x(x-1)(x+1)}$$

$$137. \lim_{x \rightarrow \infty} \frac{2x^3 + 7x - 5}{x^2 - x}$$

$$138. \lim_{x \rightarrow \infty} \frac{2x^2 + x}{\sqrt{6x + 3x^3}}$$

$$139. \lim_{x \rightarrow \infty} \frac{\sqrt{2x + x^4}}{2x - 3}$$

$$140. \lim_{x \rightarrow \infty} \frac{9x^4 + x}{x^2 - x^3}$$

$$141. \lim_{x \rightarrow \infty} \frac{3x^2 - 5}{2x^3 + x - 1}$$

$$142. \lim_{x \rightarrow \infty} \frac{3x + 5}{2x^2 + 4x + 5}$$

$$143. \lim_{x \rightarrow \infty} \frac{3x^2 + 5x - 7}{10x^3 + 5x}$$

$$144. \lim_{x \rightarrow \infty} \frac{x^2 - 17}{\sqrt{x^6 + 5x^3} - 5}$$

$$145. \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 5x - 1}}{3x^2 - 9}$$

$$146. \lim_{x \rightarrow \infty} \left( \frac{\sqrt{x+4} - \sqrt{2x+1}}{x-3} \right)$$

$$147. \lim_{x \rightarrow \infty} \frac{x^2 - 17}{\sqrt{x^6 + 5x^3 - 5} + \sqrt{3x^6 - 2}}$$

$$148. \lim_{x \rightarrow \infty} \frac{x - 2}{\sqrt{4x^2 - 2x - 6} - \sqrt{x^2 + 1}}$$

$$149. \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 5x - 1}}{\sqrt{3x^4 - 9x + 1}}$$

$$150. \lim_{x \rightarrow \infty} (\sqrt{x+6} - \sqrt{x+3})$$

$$151. \lim_{x \rightarrow \infty} (\sqrt{x+3} - \sqrt{x+2})$$

$$152. \lim_{x \rightarrow \infty} (\sqrt{2x-1} - \sqrt{x+4})$$

$$153. \lim_{x \rightarrow \infty} (\sqrt{4x+2} - \sqrt{x-3})$$

$$154. \lim_{x \rightarrow \infty} (\sqrt{x+5} - \sqrt{x})$$

$$155. \lim_{x \rightarrow \infty} (\sqrt{3x+1} - \sqrt{3x-1})$$

$$156. \lim_{x \rightarrow \infty} (\sqrt{x+1} - 2\sqrt{x-3})$$

$$157. \lim_{x \rightarrow \infty} (3\sqrt{x+6} - 2\sqrt{1-x})$$

$$158. \lim_{x \rightarrow \infty} (\sqrt{ax+b} - \sqrt{px+q})$$

untuk:  $a = p$ ,  $a > p$  dan  $a < p$

$$159. \lim_{x \rightarrow \infty} (\sqrt{x^2 + x + 1} - \sqrt{2x^2 + x})$$

$$160. \lim_{x \rightarrow \infty} (\sqrt{4x^2 + 6x - 1} - \sqrt{5x^2 - x + 9})$$

$$161. \lim_{x \rightarrow \infty} (\sqrt{x^2 + 2x - 1} - \sqrt{(x-2)(2x+9)})$$

$$162. \lim_{x \rightarrow \infty} (\sqrt{4x^2 - 5} - \sqrt{x^2 - 3x})$$

$$163. \lim_{x \rightarrow \infty} (\sqrt{2x^2 + x - 5} - \sqrt{x^2 - 3x + 12})$$

164.  $\lim_{x \rightarrow \infty} \left( \sqrt{(3x+1)(x-5)} - \sqrt{x^2 + 7x + 1} \right)$
165.  $\lim_{x \rightarrow \infty} \left( \sqrt{(3x-5)(x+4)} - \sqrt{3x^2 - 7x + 1} \right)$
166.  $\lim_{x \rightarrow \infty} \left( x - \sqrt{4x^2 - 7x - 1} \right)$
167.  $\lim_{x \rightarrow \infty} \left( (x+2) - \sqrt{4x^2 - 7x + 8} \right)$
168.  $\lim_{x \rightarrow \infty} \left( x + 5 - \sqrt{x^2 - x - 9} \right)$
169.  $\lim_{x \rightarrow \infty} \left( (x+3) - \sqrt{(x-3)(x+3)} \right)$
170.  $\lim_{x \rightarrow \infty} \left( \sqrt{3x^2 + 3x - 5} - x + 4 \right)$
171.  $\lim_{x \rightarrow \infty} \left( \sqrt{x^2 + 6x + 5} - x - 4 \right)$
172.  $\lim_{x \rightarrow \infty} \left( \sqrt{x^2 - 1} - 2x - 3 \right)$
173.  $\lim_{x \rightarrow \infty} \left( \sqrt{4x^2 + 3x - 5} - (2x - 3) \right)$
174.  $\lim_{x \rightarrow \infty} \left( \sqrt{9x^2 + x - 4} - (3x + 5) \right)$
175.  $\lim_{x \rightarrow \infty} \left( \sqrt{2x^2 - 3x + 5} \right)$
176.  $\lim_{x \rightarrow \infty} \left( \sqrt{x^2 - 3x} - \sqrt{2x^2 + 8} \right)$
177.  $\lim_{x \rightarrow \infty} \left( \sqrt{3x - \sqrt{x}} - 4 - \sqrt{3x + 2\sqrt{x} - 5} \right)$
178.  $\lim_{x \rightarrow \infty} \left( \sqrt{4x^4 + 3x^2 - 1} - \sqrt{4x^4 + 5x^2 + 1} \right)$
179.  $\lim_{x \rightarrow \infty} \left( \sqrt{x^3 - 4} - \sqrt{x^3 + 8} \right)$
180.  $\lim_{x \rightarrow \infty} \left( \frac{\sqrt{1 + 4x^2} - \sqrt{1 + 9x^2}}{x} \right)$
181.  $\lim_{x \rightarrow \infty} \left( \frac{\sqrt{1 + x^2} - \sqrt{4 + x^2}}{x} \right)$
182.  $\lim_{x \rightarrow \infty} \left( x \left( \sqrt{x^2 + 2} - x \right) \right)$
183.  $\lim_{x \rightarrow \infty} \left( \frac{4}{x^2} - \frac{3}{x} + 2 \right)$
184.  $\lim_{x \rightarrow \infty} \left( \frac{x+3}{2x-1} - \frac{2x+5}{x-7} \right)$
185.  $\lim_{x \rightarrow \infty} \left( \frac{-4x^2}{2x^2 + 9x - 5} - \frac{3x}{x+5} \right)$
186.  $\lim_{x \rightarrow \infty} \frac{\sqrt[3]{x^2 + x^3} \sqrt{x}}{x^2}$  \*\*
187.  $\lim_{x \rightarrow \infty} \frac{3\sqrt[3]{x^2 - x^2} \sqrt{x}}{6 + x^2 \sqrt{x}}$  \*\*

**Hitunglah nilai dari limit fungsi berikut:**

188.  $\lim_{x \rightarrow \frac{\pi}{2}} \sin x + 5 \cos x$
189.  $\lim_{x \rightarrow 0} (\sin 2x \cdot \cot x)$
190.  $\lim_{x \rightarrow \frac{\pi}{2}} \left( \frac{\sin x}{6} + \frac{5 \cos x}{3 \sin x} \right)$
191.  $\lim_{x \rightarrow 0} \frac{\cos x}{2x}$
192.  $\lim_{x \rightarrow 0} \frac{x+5}{\cos x}$
193.  $\lim_{x \rightarrow 0} \frac{\tan 2x}{\sin 5x}$
194.  $\lim_{x \rightarrow 0} \frac{\sin 3x}{5x}$
195.  $\lim_{x \rightarrow 0} \frac{x \sin 5x}{\sin^2 3x}$

$$196. \lim_{x \rightarrow 0} \frac{\tan^2 \frac{1}{2} x}{\sin 3x \sin 2x}$$

$$197. \lim_{x \rightarrow 0} \frac{2x^2}{\sin^2 x}$$

$$198. \lim_{x \rightarrow 0} \frac{\sin^2 3x}{(3x)^2}$$

$$199. \lim_{x \rightarrow 0} \frac{\tan 2x}{x \sec 2x}$$

$$200. \lim_{x \rightarrow 0} \frac{x}{\sin \frac{x}{2} \cos \frac{x}{2}}$$

$$201. \lim_{x \rightarrow 0} \frac{2x}{\cos x}$$

$$202. \lim_{x \rightarrow 0} \frac{\sin^2 2x}{x^2}$$

$$203. \lim_{x \rightarrow 0} \frac{\cos x - \cos 3x}{x^2}$$

$$204. \lim_{x \rightarrow 0} \frac{\sin 3x + \sin 4x}{x}$$

$$205. \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x}$$

$$206. \lim_{x \rightarrow p} \frac{1 + \cos x}{\sin 2x}$$

$$207. \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{2x^2}$$

$$208. \lim_{x \rightarrow 0} \frac{\sin(2x^2)}{x^2 + \sin^2 3x}$$

$$209. \lim_{x \rightarrow 0} \frac{\sin 4x \tan^2 3x + 6x^3}{2x^2 \sin 3x \cos 2x}$$

$$210. \lim_{x \rightarrow a} \frac{\cos x - \cos a}{x - a}$$

$$211. \lim_{x \rightarrow 0} \frac{\cos x - \cos 3x}{1 - \cos x}$$

$$212. \lim_{x \rightarrow 0} \frac{\cos 5x - \cos 9x}{1 - \sqrt{\cos x}}$$

$$213. \lim_{x \rightarrow \frac{p}{4}} \frac{\cos 2x}{p - 4x}$$

$$214. \lim_{x \rightarrow \frac{p}{4}} \frac{\cos^2 x}{1 - \sin x}$$

$$215. \lim_{x \rightarrow 1} \frac{(x^3 - 1)^{\frac{1}{2}} \tan(x-1)}{(x^2 - 1)^{\frac{1}{2}} \sin(x-1)}$$

$$216. \lim_{x \rightarrow 1} \frac{(x^2 - 1) \sin 2(x-1)}{-2 \sin^2(x-1)}$$

$$217. \lim_{x \rightarrow 0} \frac{1 - \cos x}{x \sin x}$$

$$218. \lim_{x \rightarrow \frac{p}{2}} (\sec x - \tan x)$$

$$219. \lim_{x \rightarrow 0} \frac{\sin x - \tan x}{x^3}$$

$$220. \lim_{x \rightarrow y} \frac{(3x + 3y) + \tan(x + y)}{9x + 9y}$$

$$221. \lim_{x \rightarrow 0} (x \cot 2x)$$

$$222. \lim_{x \rightarrow 1} \frac{x - 1}{\tan px}$$

$$223. \lim_{x \rightarrow \frac{p}{4}} \frac{\tan x - 1}{\cos 2x}$$

$$224. \lim_{x \rightarrow \frac{p}{4}} \frac{\cos 2x}{x(\tan x - 1)}$$

$$225. \lim_{x \rightarrow 0} \frac{\sin 2x}{3 - \sqrt{2x + 9}}$$

$$226. \lim_{x \rightarrow 0} \frac{\sin 4x}{1 - \sqrt{1 - x}}$$

$$227. \lim_{x \rightarrow 1} \frac{\sin\left(1 - \frac{1}{x}\right) \cos\left(1 - \frac{1}{x}\right)}{x - 1}$$

$$228. \lim_{x \rightarrow 2} \frac{\sin(x - 2)}{x - 2}$$



229.  $\lim_{x \rightarrow p} \frac{\sin(x-p)}{x-p}$
230.  $\lim_{x \rightarrow 1} \frac{(3x+1)\sin(x-1)}{x^2+2x-3}$
231.  $\lim_{x \rightarrow 3} \frac{\sin(\sqrt{x+1}-2)}{x-3}$
232.  $\lim_{x \rightarrow \frac{p}{2}} \frac{1-\sin x}{\frac{p}{2}-x}$
233.  $\lim_{x \rightarrow \frac{p}{4}} \frac{\sin x - \sin \frac{p}{4}}{x - \frac{p}{4}}$
234.  $\lim_{x \rightarrow \frac{p}{2}} \frac{2 \tan x}{\sec x}$
235.  $\lim_{x \rightarrow 0} \frac{\tan 2x \cdot \tan 3x}{5x^2}$
236.  $\lim_{x \rightarrow 0} \frac{1+\cos x}{1+\sin x}$
237.  $\lim_{x \rightarrow 0} \frac{1-\cos 2x}{1-\cos x}$
238.  $\lim_{x \rightarrow 0} \frac{x^2+3x}{\sin x}$
239.  $\lim_{x \rightarrow 0} \frac{2x^2}{1-\cos^2 \frac{1}{2}x}$
240.  $\lim_{x \rightarrow 0} \frac{\sin 3x - \sin 3x \cdot \cos 2x}{4x^3}$
241.  $\lim_{x \rightarrow 2} \frac{(x^2-5x+6)\sin(x-2)}{(x^2-x-2)^2}$
242.  $\lim_{x \rightarrow 0} \frac{(x^2-1)\sin 6x}{x^3+3x^2+2x}$
243.  $\lim_{x \rightarrow 0} \frac{\sin 8x + \sin 2x}{4x \cos 3x}$
244.  $\lim_{x \rightarrow 0} \frac{\sin 2x}{3-\sqrt{2x+9}}$
245.  $\lim_{x \rightarrow 0} \left( \frac{\sin 5x - \sin 2x}{\sin 8x - \sin 3x} \right)$
246.  $\lim_{x \rightarrow 0} \left( \frac{\tan 2x - \tan x}{\sin 2x - \sin x} \right)$
247.  $\lim_{x \rightarrow \frac{p}{4}} \left( \frac{1 - \tan x}{\frac{p}{4} - x} \right)$
248.  $\lim_{x \rightarrow \frac{p}{2}} \left( \frac{1 - \cos 4x}{x \sin x} \right)$
249.  $\lim_{x \rightarrow \frac{p}{2}} \left( \frac{\sin(\cos x)}{\cos x} \right)$
250.  $\lim_{x \rightarrow \frac{1}{4}p} \frac{\cos x - \sin x}{x - \frac{1}{4}p}$
251.  $\lim_{x \rightarrow \frac{p}{3}} \frac{\sin\left(3x + \frac{p}{3}\right) + \sin\left(x - \frac{4p}{3}\right)}{2x + \frac{2p}{3}}$
252.  $\lim_{x \rightarrow \frac{1}{2}p} \frac{\sin x - \cos x}{1 - \sin 2x}$
253.  $\lim_{x \rightarrow 1} \frac{\sin(x^2-1)}{x-1}$
254.  $\lim_{x \rightarrow \frac{1}{2}p} \frac{1+\cos 2x}{\cos x}$
255.  $\lim_{x \rightarrow a} \frac{3(x-a)}{\sin(x-a)+2x-2a}$
256.  $\lim_{x \rightarrow 1} \frac{x^3 - (a+1)x^2 + ax}{(x^2-1) + \tan(x-1)}$
257.  $\lim_{x \rightarrow p} \frac{1+\cos x}{x-p}$
258.  $\lim_{x \rightarrow 0} \frac{\sin 2x(1+\cos x)}{\tan x(1+3\sec x)}$
259.  $\lim_{x \rightarrow 0} \frac{3\sin 2x - 2\sin 3x}{x(1-\cos 3x)}$

$$260. \lim_{x \rightarrow 0} \frac{x^3}{\sin 2x - \tan 2x}$$

$$261. \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$$

$$262. \lim_{x \rightarrow 3} \frac{1 - \cos(x+3)}{x^2 + 6x + 9}$$

$$263. \lim_{x \rightarrow a} \frac{1 - \sqrt{1 - \sin^2(x-a)}}{(x-a) \tan 5(x-a)}$$

$$264. \lim_{x \rightarrow 3} \left( \frac{x-3}{x - \sin(x-3) - 3} \right)$$

$$265. \lim_{x \rightarrow 0} \frac{\sin 2x + \sin 6x + \sin 10x - \sin 18x}{3 \sin x - \sin 3x}$$

$$266. \lim_{x \rightarrow y} \left( \frac{\tan x - \tan y}{\frac{x}{y} - 1 + \left( \frac{x}{y} - 1 \right) \tan x \tan y} \right) **$$

**Tentukan, jika ada, titik-titik yang menyebabkan fungsi-fungsi berikut tidak kontinu:**

$$267. f(x) = \frac{x^2 - 1}{x^2 + x}$$

$$268. f(x) = \frac{x}{\sqrt{2x-3}}$$

$$269. f(x) = \frac{x^2 - 4}{\sqrt{x^2 - 3x + 2}}$$

$$270. f(x) = \frac{x^2 + 2x + 3}{x^3 - 1}$$

$$271. f(x) = \frac{2x^2 - 5x - 3}{x^2 + x - 2}$$

$$272. f(x) = \frac{x^2 + 1}{x^2 + 3x - 10}$$

$$273. f(x) = \frac{2x+1}{x^2 - x + 1}$$

$$274. f(x) = \begin{cases} 1; & \text{unt } x < 0 \\ 1-x; & \text{unt } x \geq 0 \end{cases}$$

$$275. f(x) = \begin{cases} 2x; & \text{unt } x < 0 \\ -x; & \text{unt } x \geq 0 \end{cases}$$

$$276. f(x) = \begin{cases} x; & \text{unt } x < 0 \\ 1; & \text{unt } x = 0 \\ x^2; & \text{unt } x > 0 \end{cases}$$

$$277. f(x) = \frac{x^2 - 1}{x - 1}$$

$$278. f(x) = \begin{cases} \frac{x^2 - 1}{x - 1}; & \text{unt } x \neq 1 \\ 2; & \text{unt } x = 1 \end{cases}$$

**Selidikilah, apakah fungsi-fungsi berikut kontinu pada titik yang diberikan:**

$$279. f(x) = 5, \text{ pada } x = 1$$

$$280. f(x) = 5x - 10, \text{ pada } x = -3$$

$$281. f(x) = \frac{8}{x-3}, \text{ pada } x = 3$$

$$282. f(x) = \frac{1}{x^2 - x - 6}, \text{ pd } x = 3 \text{ dan } x = -2$$

283.  $f(x) = \frac{3x-12}{x^2-7x+12}$ , pada  $x = 4$

284.  $f(x) = \frac{3x^2+3x-6}{2x^2-2x-12}$ , pada  $x = -2$

**Hitunglah nilai dari limit fungsi berikut:**

285.  $\lim_{x \rightarrow \infty} \left( \frac{x}{x+1} \right)^{x+1}$

291.  $\lim_{x \rightarrow \infty} \left( \frac{3x+1}{3x+5} \right)^{2x+3}$

286.  $\lim_{x \rightarrow \infty} \left( 1 + \frac{2}{3+x} \right)^{-2x}$

292.  $\lim_{x \rightarrow \infty} \left( \frac{2-5x}{1-5x} \right)^{2x+3}$

287.  $\lim_{x \rightarrow \infty} \left( \frac{x+5}{x+3} \right)^{x+6}$

293.  $\lim_{x \rightarrow \infty} \left( \frac{6x+5}{6x-1} \right)^{7x+2}$

288.  $\lim_{x \rightarrow \infty} \left( \frac{2x+2}{2x+6} \right)^{2x}$

294.  $\lim_{x \rightarrow \infty} \left( \frac{x^2+3x+2}{x^2+5x+1} \right)^{\frac{x^2+1}{x+1}}$

289.  $\lim_{x \rightarrow \infty} \left( 1 + \frac{a}{x} \right)^x$

290.  $\lim_{x \rightarrow \infty} \left( 1 + \frac{1}{x} \right)^{ax}$

**Hitunglah nilai dari  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$  dari fungsi-fungsi berikut:**

295.  $f(x) = 9$

296.  $f(x) = 5x$

297.  $f(x) = 8x - 10$

298.  $f(x) = x^2$

299.  $f(x) = 3x^2$

300.  $f(x) = -2x^2 + 1$

301.  $f(x) = 2x^2 + 3x$

302.  $f(x) = x^3$

303.  $f(x) = 2x^3$

304.  $f(x) = \sqrt{x}$

305.  $f(x) = 2\sqrt{x}$

306.  $f(x) = 2\sqrt{x+1}$

